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Pythagorean Music Theory and Its Application in Renaissance Architectural Design

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Abstract

This article concerns Alberti's approach to architectural design integrates Pythagorean consonance ratios from music theory, not by direct application but as a conceptual guide for aesthetic ratios in buildings. He expands spatial dimensions using methods derived from past artisans, adhering to and building upon the foundational consonance ratios. While applying these two-number proportions to width and length poses no problem, calculating height in a three-dimensional space requires a three-number proportion. Alberti resolves this issue by adopting the mean value strategy from music theory, where the mean is typically the middle value in an octave ratio, to determine the height in the spatial configuration. Alberti advocates for architects to leverage the insights of skilled artisans. However, his own architectural work demonstrates deviations from his theoretical principles. This study highlights Alberti's application of musical intervals such as the major and minor thirds, sixths, and sevenths, which were not traditionally emphasized in the musical science of the quadrivium, focused primarily on octaves, fifths, and fourths. These choices reflect the broader evolution of music during the 15th century, marking a shift towards "practical music" and signaling music's departure from the liberal arts to become a significant element of the fine arts, showcasing the influence of Pythagorean consonance in European music and arts through the 16th century. Thus, this article reflects the interactions between music and architecture of knowledge received from ancient Greek civilization in the Renaissance. The art of music in this period is founded on the philosophy and knowledge of Pythagoras.

Keywords: application, architectural design, music theory, Pythagorean principle, Renaissance

Introduction

The fifteenth century witnessed a fundamental transformation in European history as the Renaissance revolutionized economic, political, and social frameworks. The era celebrated humanism, which values human capabilities and replaced the previously dominant belief in the divine as the focal point of the universe. An event that catalyzed changes in the arts and sciences was the fall of Constantinople in 1453, which led to the transportation of manuscripts containing ancient Greek knowledge across various disciplines to Italy. This event provided an incentive for humanists to immerse themselves in the study of original Greek texts, allowing for comparative analyses with Latin translations that dated back 500 years. The event prompted a series of philosophical debates, prominently featuring contributions from the circles of Plato and Aristotle. A notable theme in these discussions was the interpretation of Pythagoras' concept of geometric proportions, which in turn birthed the emergence of the New Pythagoreans, Neoplatonism, and New Aristotelianism. The discourse held in Florence among the philosophers of the Florentine Camerata[1] was instrumental in revolutionizing classical music in Italy. The interpretation of music underwent a significant transformation under the philosophical frameworks of Plato and Aristotle. Plato regarded music as an educational institution, emphasizing its role in guiding individuals into the realm of morality. He prioritized the text in vocal music, reflecting his belief in the profound impact of lyrical content over instrumental accompaniment. In contrast, Aristotle adopted a more liberal stance, viewing music not merely as a vehicle for moral instruction but also as a source of enjoyment.[2] This inclusivity extended to instrumental music, which he believed could convey textual significance as effectively as vocal elements. The divergent views of these two schools spurred intellectual movements that explored the relationship between music and rhetoric, which eventually evolved into discussions on music and drama, culminating in the development of opera[3]. This progression underscores a reaction against the dominance of vocal polyphony, characteristic of the Northern school from the Netherlands, emphasizing a return to text-centric musical forms. Furthermore, it is critical to recognize that the Florentine Camerata uncovered significant misinterpretations in the music theory system that Boethius had translated from Nichomachus. Their findings revealed that the ancient Greek scale, which noted downward movements and initiated modes with Dorian on e, Phrygian on d, and Lydian on c, bore no correlation to the ecclesiastical modes, shedding light on historical inaccuracies in the theoretical understanding of music.[4]

Historically, church architects and artisans, who devoted their crafts to serving Christianity, adhered strictly to repetitive geometric patterns, employing a templated approach in both content and composition. Their social standing was limited to that of craftsmen, organized within guild societies according to their specific trades. However, as the popularity of Gothic architecture and vocal polyphony—both hallmarks of regions north of the Alps—began to wane in Italy, a shift occurred.[5] Architects and artisans began to assert their creativity more vigorously, eventually transcending their traditional roles and gaining recognition as true artists. This evolution coincided with significant socio-economic changes during the 15th century, particularly in Florence, where affluent merchant families, most notably the Medici, commissioned architects to construct lavish residences known as palazzi. These families also sponsored the creation of paintings and sculptures to reflect their status. Consequently, artistic styles in both architecture and the visual arts began to shift from religious to secular themes. Artists and architects explored new subjects, such as Greek deities, often represented as emblems of the virtues of patrician families. In addition, Renaissance artists and architects drew inspiration from Pythagorean principles, applying the concepts of consonance and proportion to their creations. For instance, artists like Pomponius Gauricus (1481/82-1530)[6] adopted Pythagorean ratios in anatomical studies, while architects such as Leon Battista Alberti incorporated these principles into the design of city palazzi in Florence, aiming to mirror the harmonic

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[1] See Claude V. Palisca, *The Florentine Camerata: Documentary Studies and Translation*, Yale University Press, New Haven, 1989.

[2] Rolf Dammann, *Der Musikbegriff im deutschen Barock*, Laaber-Verlag, Laaber, 1984, p. 97 and Susanne Schaal, Musica Scenica: Die Operntheorie des Giovanni Battista Doni, (Peter Lang) Frankfurt am Main 1993, pp. 37ff.

[3] Eggebrecht, Hans Heinrich. "Monodie." *Musik im Abendland*. Piper, 1991, pp. 338ff.

[4] Claude V. Palisca, *Humanism in Italian Renaissance Musical Thought*, (Yale University Press) New Haven 1985, p. 280.

[5] The influence of these factors began to play a role in Italy during The Great Schism (1378-1417). See Reinhard Strohm, *The Rise of European Music*, 1380-1500, (Cambridge University Press, 1993).

[6] Pomponius Gauricus, *De Sculptura*, with notes and translation by André Chastel and Robert Klein, from the edition published by P. de Giunta, Florence, 1504, Librairie Droz, Paris, 1969. Also see Wittkower, Rudolf. Architectural Principles in the Age of Humanism. Alec Tiranti, 1952, p. 104.

proportions found in music. This reorientation established a new aesthetic that combined mathematical precision with classical ideals of beauty.

In the Renaissance, Florence was recognized as a center for philosophy, academic pursuits, literature, art, and music. This era was characterized by an intellectual shift towards "scientia," a science-based approach grounded in numerical theory. This theoretical framework, which referenced Pythagoras, was particularly influential in the composition and theory of practical music.

This text provides an interpretive analysis that aims to highlight the impact of the Pythagorean concept of consonance, a principle from ancient Greek theory, as it is reflected in Renaissance artwork and architecture. This study examines the adoption of the beauty ratio, a mathematical construct from music theory, and its application to the design principles of the period's architecture.

The Interpretation of Raphael's "The School of Athen"[7] (1510-11)

Raphael's fresco, "The School of Athens," portrays pivotal philosophers from various Greek schools of thought, including Plato and Aristotle, who are centrally positioned. Plato is shown holding his work "Timaeus," while Aristotle holds "Nicomachean Ethics." In the composition's lower left, Pythagoras is depicted writing, and Parmenides holds a tablet displaying two sets of numbers that are crucial to understanding music, art, and architecture (see Figure 1).

Figure 1. Fresco "The School of Athens" by Raphael (1510-11) in Apostoclic Palace, Vatican City

The two sets are as follows:

The upper set of numbers—6 (VI), 8 (VIII), 9 (IX), and 12 (XII)—are arranged horizontally, while the lower set—1 (I), 2 (II), 3 (III), 4 (IV)—forms a vertical pyramid, with the number 10 (X) beneath, symbolizing the sum of 1+2+3+4.

Each set of numbers is referred to as a "tetractys." (see Figure 2)

[7] Raphael, or Raffaello Sanzio da Urbino (1483-1520), was commissioned by Pope Julius (r. 1503-13) to paint frescoes in the papal suite at the Vatican Palace. These are now known as the "Stanza di Raffaello." For an overview of the history of the paintings and the program of figures depicted, see Ingrid Rowland, "The Vatican Stanze," in *The Cambridge Companion to Raphael*, edited by Marcia B. Hall (Cambridge University Press, 2005), pp. 95-119.

Figure 2. Tablet of Pythagoras in the fresco School of Athens by Raphael

Additionally, the tetractys numbers 6, 8, 9, and 12 are interconnected using a curved line, illustrating key concepts from Greek music theory—the diatessaron, diapente, and diapason. The largest curved line, framing the numbers 6 and 12, represents the diapason, which in music theory corresponds to the octave interval, equating to a ratio of 6:12. The curved line connecting 6-9 and 8-12 illustrates the diapente, equivalent to a 2:3 ratio (6:9 and 8:12), mirroring the fifth interval. The smallest curved line, linking 6-8 and 9-12, denotes the diatessaron, corresponding to a 3:4 ratio (6:8 and 9:12), akin to the fourth interval. Furthermore, the ratio 8:9, labeled as epogdoon, compares to a whole tone[8], dividing the interval of the consonant fourth interval (6:8 and 9:12) and is integral to the structure of the Greek tetrachord system. This ratio is derived from the measurement of a monochord. In terms of numeric progression, the lower group of numbers 1, 2, 3, and 4, arranged vertically in a pyramidal formation, represents the cumulative sum of the series. Positioned at the base of this arrangement is the number 10 (X), considered a symbol of completeness in the Middle Ages.

Understanding the origin of the tetractys, a concept rooted in Pythagorean harmonic progression, necessitates an exploration of Pythagoras's impact on the theory of consonance.[9] During the eras of ancient Greece and the Middle Ages, Pythagoras founded the discipline of "ars musica", a crucial element of the medieval quadrivium, which also included arithmetic, geometry, and astronomy. it is a theoretical, almost entirely mathematical study of musical tones and their relationships. He identified the consonance ratios, known as "symphone" in Greek, which encompass the octave, fifth, fourth, double octave, and octave-plus-fifth. These intervals are theoretically expressible through numbers and ratios, which are central to the disciplines of arithmetic and music within the quadrivium, in contrast to the trivium disciplines, which are based on the rules of the language and the grammatical rules that govern its use. It is important to recognize

[8] "Epogdoon" is a term in Greek mathematics meaning a mixed number consisting of the integer 1 and the fraction 1/8 (1 1/8), or the improper fraction 9/8, which in Greek music theory denotes a whole tone. See Julius Schwabe, *Hans Kayser's Last Discovery: The Pythagorean Tetractys in Raphael's "School of Athens"*, in: *Symbolon*, Vol. 5, Schwabe & Co., Basel, 1966, pp. 92-102, especially p. 97.

[9] Boethius, De Institutione musica, Book 2, Chapter 2, ดูฉบับแปลภาษาอังกฤษ Palisca, Claude V. (1989). *Fundamentals of Music: Anicius Manlius Serverius Boethius*. translation, with Introduction and Notes by Calvin M. Bower. Yales University Press New Haven 1989, pp. 52ff.

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that music theory during the Greek era and Middle Ages was a speculative science rather than a form of practical music. This field of study, considered a branch of philosophy, aimed to discover the underlying principles and natural phenomena in the universe and on Earth. 'Sound' was thus regarded as a measurable and tangible phenomenon that could be quantified through numerical ratios. Over time, music adopted these arithmetic-based rules.[10]

The elements of consonance in music are deeply rooted in the principles of mathematics, specifically the ratios defining various musical intervals: the octave, also called diapason (1:2), the fifth or diapente (2:3), the fourth known as diatessaron (3:4), the double octave or bis diapason (1:4), and the octave-plus-fifth or diapente ac diaspason (1:3). These ratios, when arranged, form a 1, 2, 3, and 4 sequence, known as the tetractys in Greek tradition. This numerical arrangement underpins the science of music, linking numbers to musical tones. For instance, the octave consists of a fifth and a fourth (2/3 multiplied by 3/4 equals 1/2), while the octave-plus-fifth includes an octave and a fifth (1/2 multiplied by 3/2 equals 1/3).

In Greek music theory, numbers define the structure of the two-octave tone system, known as the systema teleion[11], which includes[12]:

4	a´[13]	Nete hyperbolaio
3	e´	Nete dieszeugmenon
2	a	Mese
1	A	Proslambanomenos

If we adjust Nete dieszeugmenon (e´) downward within the octave system defined by the ratios 2:3:4, we arrive at Hypate meson (e), which is centrally located within the octave spanning from A to a. This adjustment results in the formation of an octave from e´ to e, structured in the Dorian mode on E. The adjusted scale is as follows:

4	a´	Nete hyperbolaio	
3	e´	Nete dieszeugmenon	
2	a	Mese	4
4	e	Hypate meson	3
1	A	Proslambanomenos	2

[10] *Boethius and the Liberal Arts: A Collection of Essays*, edited by Michael Masi, (Peter Lang) Las Vegas 1981, p. 13.
[11] For details on Greek theoretical systems, see "The State of Knowledge of Greek Theory," in Claude V. Palisca, *Humanism in Italian Renaissance Musical Thought*, pp. 35-50. Also refer to Frieder Zaminer, "Harmonik und Musiktheorie im Alten Griechenland," in *Geschichte der Musiktheorie*, Vol. 2, Wissenschaftliche Buchgesellschaft, Darmstadt, 2006, pp. 47-256, especially pp. 156-171.
[12] This procedure is based on the method used by Barbara Münxelhaus, Pythagoras musicus: Zur Rezeption der pythagoreischen Musiktheorie als quadrivialer Wissenschaft im lateinischen Mittelalter, (Verlag für systematische Musikwissenschaft) Bonn-Bad Godesberg 1976, pp. 18-20.
[13] Comparison of the Romanic alphabet in the central column with modern musical notes for easier understanding.

If we consider the octave from e´ to e and adjust it to match the structure of the octave from a´ to a, we would introduce Paramese (b) into the sequence. The restructured scale would then appear as follows:

4	a´	Nete hyperbolaio		
3	e´	Nete dieszeugmenon		3
	b	Paramese		4
2	a	Mese	4	2
	e	Hypate meson	3	
1	A	Proslambanomenos	2	

In the Dorian octave from e´ to e, there is a distinct characteristic involving two segments. The segment e-b-e´ consists of a fifth and a fourth, while e-a-e´[14] consists of a fourth and a fifth.

	e´	Nete dieszeugmenon e´		
3/2				4/3
	b	Paramese		
		Mese	a	
4/3				3/2
	e	Hypate meson	e	

Reorganizing the notes of a two-octave scale (1:4 ratio) to expand from a base of 4 results in the following configuration as illustrated in Gaffurio's "Theorica Musice", Series of six numbers; 4, 6, 8, 9, 12, 16 (see Figure 3):

16	a´	Nete hyperbolaio
12	e´	Nete dieszeugmenon
9	b	Paramese
8	a	Mese
6	e	Hypate meson
4	A	Proslambanomenos

[14] In the Greek tone system and scale, notes descend from high to low. The Western theory of modes for the first 500 years was based on a misunderstanding of this Greek system, originating from Boethius's *De institutione musica*. This treatise, which translated and commented on Nikomachus's music theory, served as a standard text since the Middle Ages. However, Boethius misinterpreted some content, leading to errors. Renaissance theorists revisited the original Greek manuscripts and realized that the structure of the plainchant modes was unrelated to those of the ancient Greeks, merely borrowing their names. For more details, see "Chapter Eleven: Greek Tonality and Western Modality," in Claude V. Palisca, *Humanism in Italian Renaissance*, pp. 280-332.

Figure 3. A page from Gaffurio's *Theorica Musicae* (1492).

According to the theory of Aristides Quintilianus, the two sets of tetractys are fundamental to the Pythagorean harmonic progression. This theory establishes a connection with geometry, starting from the structure of the octave (2:4), which integrates elements of the fifth (2:3) and fourth (3:4) intervals. This relationship is mathematically represented as 2/4 = (2/3) x (3/4). Notably, these ratios—2:4, 2:3, and 3:4—correspond to the proportions of width to length in rectangles. The areas of these rectangles are computed as follows: 2x4 = 8, 2x3 = 6, 3x4 = 12, and the square of 3 calculates to 9. The culmination of these values forms the tetractys 6, 8, 9, 12.

The two sets of tetractys (1, 2, 3, 4 and 6, 8, 9, 12) differ in their musical applications as follows:

- First Set (1, 2, 3, 4): Spans two octaves and forms five consonant pairs:
  - octave: 1:2
  - fifth: 2:3
  - fourth: 3:4
  - double octave: 1:4
  - octave and a fifth: 1:3 This set illustrates how an octave can include a fifth and a fourth, with ratios adding up as 6:9 + 9:12.
- Second Set (6, 8, 9, 12): Fits within a single octave (E' to E) and includes:
  - One octave: 6:12
  - Two fifths: 6:9, 8:12

   - Two fourth: 6:8, 9:12 These intervals illustrate how an octave can also be divided into a fifth and a fourth, plus an additional fourth and a fifth.

- Tonal Importance of 8:9 (a-b): This interval is key in forming two consonant fourths (b-é and e-a), each with a diatonic structure of two whole tones and a semitone. These intervals are foundational in tetrachord construction, leading to the development of the 7 modes[15] and the modern diatonic scale. Nichomachus prioritized the second tetractys set (6, 8, 9, 12) and referred to the first set, which includes the octave (1:2), fifth (2:3), and fourth (3:4), as the "basic consonances."[16]

There is another notable correlation between music and mathematics in that proportion contains two types: proportio and proportionalitas. Proportio refers to a ratio that consists of two numerical figures—such as the octave (1:2), fifth (2:3), and fourth (3:4). Proportionalitas extends to ratios involving more than two numbers, often focusing on the middle number as the mean, or medietas. For instance, in the ratio 2:3:4, the number 3 is highlighted. This number is called the mean. Often in music theory, the mean is found in the octave ratio 2:4, derived from 2:3:4, and 6:2, from the sequences 6:8:12 and 6:9:12, which is computed from the components of a fifth and a fourth, or a fourth and a fifth, as previously illustrated.

In arithmetic science, three types of means are recognized: arithmetic, geometric, and harmonic[17], each calculated as follows:

- Arithmetic Mean: For numbers a and b, if a-m = m-b, then m = a+b/2. For example, with a = 12 and b = 6 , the mean is 6+12/2 = 9, resulting in the ratio 12:9:6.

- Geometric Mean: For a:m = m:b, m = $\sqrt{ab}$. For instance, with a = 12 and b = 3 , the mean is $\sqrt{12x3}$ = 6, giving the ratio 12:6:3.

- Harmonic Mean: m = $= \frac{2ab}{a+b}$. For example, with a = 12 and b = 6 , m = 2×6×12/6+12 = 8, resulting in the ratio 12:8:6.

Note that the proportions 2:3:4 and 6:9:12 correspond to a harmonic mean, where b = 12 and m = 8 for the ratio 6:8:12. Nichomachus suggests that proportionalitas and the mean are of minimal significance in music theory.[18]

We will explore the impact of Pythagorean consonance on architecture, focusing on the work of Leon Battista Alberti, a Renaissance architect.

The Interpretation of Leon Battista Alberti

Leon Battista Alberti, a Renaissance architect and humanist, discussed the application of music theory to architecture in chapters 5 and 6 of the 9th book of "De re aedificatoria." At the beginning of chapter 5, Alberti defined beauty as "[…] beauty and ornament […] which is so gathered and collected from the whole number and nature of the several parts, or to be imparted to each of them according to a certain and regular order, or which must be contrived in such a manner as to join and unite a certain number of parts into one

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[15] In addition to e Dorian mode, d Phrygian mode, c Lydian mode, b Mixolydian mode, a Hypodorian mode, g Hypophrygian mode, and f Hypo Lydian mode, the e Mixolydian mode is equivalent to the e Dorian mode.

[16] Barbara Münxelhaus, *Pythagoras musicus: Zur Rezeption der pythagoreischen Musiktheorie als quardivialer Wissenschaft im lateinischen Mittelalter*, (Verlag für systematische Musikwissenschaft) Bonn-Bad Goddesberg 1976, p. 38

[17] Boethius states that Pythagoras discovered the proportionalitas of arithmetica, geometrica, and harmonica, and Archytas of Tarentum, a disciple of Pythagoras, demonstrated calculations of proportion and mean.

[18] Roger Harmon, Die Rezeption griechischer Musiktheorie im römischen Reich, II. Boethius, Cassiodorus, Isidor von Sevilla, in: Geschichte der Musiktheorie, Band 2, Wissenschaftliche Buchgesellschaft Darmstadt 2006, p. 428.

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body or whole, ab an orderly and sure coherence and agreement of all those parts."[19] He terms the whole which contains number, finishing, and arrangement as "concinnitas" (or congruity)[20].

In his pursuit of beauty, Alberti emphasizes the role of numbers, particularly odd and even, which he associates with the "finishing" aspects of a structure's dimensions: width, length, and height. Drawing from Pythagorean principles, Alberti asserts,

> "[…] I am every day more and more convinced of the truth of Pythagoras's saying, that nature is sure to act consistently, and with a constant analogy in all her operations: From whence I conclude, that the same umbers, by means of which the agreement of sounds affects our ears with delight, are the very same which please our eyes and our minds. We shall therefore borrow all our rules for the finishing our proportion, form the musicians, who are the greatest master of this sort of number, and from those particular things wherein nature shews herself most excellent and complete […]"[21].

He argues that the same numerical relationships that harmonize sounds to delight the ear also please the eye and mind. Consequently, he suggests adopting rules from musicians, who expertly utilize such numbers, for architectural proportioning. Alberti further explores the foundational ratios of five musical consonances, using terms from Greek and Latin music theory[22]:

"Diapason"	["Dupla"]	octave (1:2)[23]
"Diapente"	"Sesquialtera	fifth (2:3)
"Diatessaron"	"Sesquitertia"	fourth (3:4)

Compound of Intervals

Diapason Diapente	[Triplum]	octave-plus-fifth (1:3)
Disdiapason	"Quadruple"	double octave (1:4)

Alberti encapsulates the numbers of proportion as the first series of tetractys (1, 2, 3, 4) in his architectural theories. While he does not specifically address the second series of tetractys (6, 8, 9, 12), he illustrates a practical application by dividing a rope into eight parts to achieve the whole tone proportion of 8:9. Towards the end of Chapter 5, Alberti states, "Of all these numbers the architects made very convenient use, taking them sometimes two by two, as in plaining our their squares and open areas, wherein only two proportions were to be considered, namely length and breath; and sometimes taking them three by three; as in public halls, council-chambers, and the like; wherein as the length was to beat a proportion to breadth, so they made the height in a certain harmonious proportion to them both."[24]

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[19] Leon Battista Alberti, De re aedificatoria, Book IX, Cap. 5, p. 194. (hereinafter cited as "De re aedificatoria") This quotation is from the English translation of The Books on Architecture by Leone Battista Alberti, originally translated into Italian by Cosimo Bartoli and into English by James L. Leoni, Venetian architect. Edited by Joseph Rykwert, Alec Tiranti, London, 1965.
[20] De re aedificatoria, Book IX, Cap. IX, Sec. 5, p. 194.
[21] De re aedificatoria, Book IX, Cap. IX, Sec. 5, p. 196f.
[22] Alberti, Leon Battista. *De Re Aedificatoria*. Book IX, Cap. IX, Sec. 5, p. 197. Also compare Boethius, Anicius Manlius Severinus. *De Institutione Musica. Book I*, Chapter 16, translated with an introduction and notes by Calvin M. Bower, edited by Claude V. Palisca, Yale University Press, 1989, pp. 22-26.
[23] The term in the third column is currently used in music theory.
[24] *De re aedificatoria*, Book IX, Cap. IX, Section 5, p. 197.

Alberti applies Pythagorean principles of consonance to architectural design by calculating room dimensions using "proportio" and "proportionalitas."[25] He categorizes spaces into three sizes—small, medium, and large—and two shapes: squares and rectangles. Specifically, rectangles are designed in 2x3 and 3x4 ratios, mirroring the musical consonances of the fifth and fourth. For instance, extending the width of a 2x3 rectangle results in a 4:6 ratio, which can be further expanded to 6:9 (4:6:9). Consequently, the central space of a 2x3 rectangle expands to a 4:9 ratio, and the dimensions of a 3x4 rectangle extend to a 9:16 ratio (9:12:16).

Alberti outlines three methods for expanding spaces into medium and large sizes:

1. Doubling all dimensions, e.g., transforming a 2x2 square into a 4x4.
2. Doubling the proportion of rectangles:
    - A 2:3 ratio doubles to 4:6, leading to a 4:9 outcome.
    - A 3:4 ratio doubles to 9:12, resulting in 9:16.
3. Expanding consonance ratios in rectangles:
    - A 1x2 rectangle (octave) expands to 2x3, achieving a 1:3 ratio (octave-plus-fifth).
    - A 1x3 rectangle expands into 2:5 using a 2:4:6 ratio or a 2:3:6 ratio.
    - A 1x4 rectangle (double octave) expands to 2:8 using a 2:4:8 ratio or to 3:12 with a 3:6:9:12 ratio.

Note: This technique of expanding two-dimensional space, unique to the architectural methodology, has no equivalent in music theory.

Alberti, in Chapter 6, elaborates on calculating height through the proportion involving three numbers, moving beyond the basic two-number interval ratios of consonance. He resolves this by employing a technique to calculate the mean—an important arithmetic method with three variations: arithmetic, geometric, and harmonic. The mean is defined as the intermediary value between two figures. Within the context of music theory, the mean corresponds to the middle note of an octave, evident in configurations like 2:3:4 and 6:9:12, where it comprises components of fifth and fourth, or 6:8:12, which includes a fourth and fifth.[26]

Conclusion

The analysis reveals Alberti's approach to architectural design integrates Pythagorean consonance ratios from music theory, not by direct application but as a conceptual guide for aesthetic ratios in buildings. He expands spatial dimensions using methods derived from past artisans, adhering to and building upon the foundational consonance ratios of 1:2 (octave), 2:3 (fifth), and 3:4 (fourth). While applying these two-number proportions to width and length poses no problem, calculating height in a three-dimensional space requires a three-number proportion. Alberti resolves this issue by adopting the mean value strategy from music theory, where the mean is typically the middle value in an octave ratio, to determine the height in the spatial configuration. Toward the end of Chapter 6 of his treatise, Alberti advocates for architects to leverage the insights of skilled artisans. However, his own architectural work, particularly the façade of Palazzo Rucellai in Florence (1446), demonstrates deviations from his theoretical principles. This façade's study highlights Alberti's application of musical intervals such as the major and minor thirds, sixths, and sevenths,[27] which were not traditionally emphasized in the musical science of the quadrivium, focused

[25] Boethius. *De Institutione Musica*, Book 2, Chapter 2, pp. 12-17; see also *Fundamentals of Music: Anicius Manlius Severinus Boethius*, translated with an introduction and notes by Calvin M. Bower, Yale University Press, 1989, pp. 65-72.
[26] Keine, Alfons. "Die musikalischen Zahlenverhältnisse in der Architektur." Phil. Diss., Technische Hochschule Hannover, 1950, pp. 11-20.
[27] Keine, Alfons. "Die musikalischen Zahlenverhältnisse in der Architektur." Phil. Disst., Technische Hochschule Hannover, 1950. Cited in Strohmeyer, Wolfgang. "Leon Battista Alberti Schönheitsbegriff und traditionelle Entwurfsgrundlagen." Archiv für Musikwissenschaft, vol. 58, no. 3, 2001, pp. 231-260.

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primarily on octaves, fifths, and fourths. These choices reflect the broader evolution of music during the 15th century, marking a shift towards "practical music" and signaling music's departure from the liberal arts to become a significant element of the fine arts, showcasing the influence of Pythagorean consonance in European music and arts through the 16th century. Thus, this article reflects the interactions between music and architecture of knowledge received from ancient Greek civilization in the Renaissance. The art of music in this period is founded on the philosophy and knowledge of Pythagoras.

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