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SHORT COMMUNICATION

A Stability Analysis for Radiation Effects on Marangoni Convection Boundary Layer Over a Permeable Surface

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ABSTRACT

The objectives are to derive the stability analysis theoretically and to set up the stability analysis numerically for radiation effects on Marangoni convection boundary layer over a permeable surface. The stability analysis is used to determine which branch solutions are stable and physically realisable. The stability can be tested via the smallest eigenvalue. Negative smallest eigenvalue produces an initial growth of disturbance and the flow becomes unstable. In contrast, the positive smallest eigenvalue results in an initial decay of the disturbance, thus the flow is stable. The research has an implication in order to identify which solution is stable, whether the first or the second solution.

Keywords: permeable surface, dual solution, stability analysis

1. INTRODUCTION

An excellent review on the Marangoni flow has been done by Tadmor (2009). Al-Mudhaf and Chamkha (2005) have studied the effects of heat generation/absorption on thermosolutal Marangoni convection with a first-order chemical reaction in the presence of suction/injection. Magyari and Chamkha (2008) obtained exact analytical solutions for the problem of steady thermosolutal magnetohydrodynamic Marangoni boundary layer flow. The non-unique solutions of Marangoni boundary layer have been obtained by Arifin et al. (2011) where the dual solution exists if a constant exponent $\beta < 0.5$. This result is consistent with the discussion given in Golia and Viviani (1986). Hamid et al. (2011a) have extended the problem of Pop et al. (2001) in the case of permeable surface and they also found the dual solution and the velocity, temperature, and concentration profiles will decrease with suction whereas injection shows the opposite effects. Thermal radiation effect on convection is essential in high-temperature processes and has many applications such as space technology, nuclear reactor cooling systems, and geothermal engineering. There is a lot of work on boundary layer flow involving radiation such as Bataller (2008), Ishak (2010), and Hamid et al. (2011b) where the existence of thermal radiation is to reduce the heat transfer rate at the surface. Due to the existence of dual solutions in a selected range of parameters, an analysis of stability is set up in order to determine the most stable solution between the two solutions by finding the smallest eigen value (Awaludin et al., 2016). The research objectives are (1) to derive the stability analysis theoretically to radiation effects on Marangoni convection boundary layer over a permeable surface, (2) to set up the stability analysis numerically to radiation effects on Marangoni convection boundary layer over a permeable surface.

2. METHODOLOGY

Under the usual boundary layer approximation, the basic governing equations are the continuity equations:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \qquad \text{Eq. 1}$$

momentum equation:

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=u_e\frac{du_e}{dx}+v\frac{\partial^2 u}{\partial y^2}, \qquad \text{Eq. 2}$$

energy equation with the effect of radiation:

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k}{\rho c_p}\frac{\partial^2 T}{\partial y^2}-\frac{1}{\rho c_p}\frac{\partial q_r}{\partial y}. \qquad \text{Eq. 3}$$

In order to perform a stability analysis, we consider the unsteady problem. Eq. 1 holds, while Eqs. 2 and 3 are replaced by:

unsteady state of momentum equation:

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=u_e\frac{du_e}{dx}+v\frac{\partial^2 u}{\partial y^2}, \qquad \text{Eq. 4}$$

and the unsteady state of energy equation:

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k}{\rho C_p}\frac{\partial^2 T}{\partial y^2}-\frac{1}{\rho C_p}\left(\frac{\partial q_r}{\partial y}\right), \qquad \text{Eq. 5}$$

where t corresponds to time. Because of the time existence, the boundary conditions are now changed to:

$$\begin{aligned}
&t<0: u=v=0, T=T_\infty \text{ for any } x, y,\\
&t\geq 0: u=cx+L_1\frac{\partial u}{\partial y},\ v=v_w, T=T_w+D_1\frac{\partial T}{\partial y} \text{ at } y=0,\\
&u\to U_e=ax, T\to T_\infty \text{ as } y\to\infty.
\end{aligned} \qquad \text{Eq. 6}$$

where c is shrinking/stretching constant, L_1 is the velocity slip factor, v_w is velocity of suction if $v_w < 0$ or injection if $v_w > 0$, T_w is temperature of the sheet, D_1 is thermal slip factor and a is the positive straining rate parameter.

We introduce a new variable τ , where τ is a dimensionless variable for time t. The similarity variables now can be rewritten as:

$$\psi = \sqrt{a\nu}xf\left(\eta\right), \quad \theta = \frac{T - T_\infty}{T_w - T_\infty}, \quad \eta = y\sqrt{\frac{a}{\nu}}, \quad \tau = at, \qquad \text{Eq. 7}$$

where η is the similarity variable and ψ is a stream function, which is defined as:

$$u = \frac{\partial \psi}{\partial y}, \; v = -\frac{\partial \psi}{\partial x}. \qquad \text{Eq. 8}$$

Variable ψ now can be reduced to:

$$u = ax\frac{\partial f}{\partial \eta}, \; v = -\sqrt{a\nu}\, f,$$

$$\frac{\partial u}{\partial t} = a^2 x\frac{\partial^2 f}{\partial \eta\, \partial \tau}, \; \frac{\partial u}{\partial x} = a\frac{\partial f}{\partial \eta},$$

$$\frac{\partial u}{\partial y} = ax\sqrt{\frac{a}{\nu}}\frac{\partial^2 f}{\partial \eta^2}, \; \frac{\partial^2 u}{\partial y^2} = \frac{a^2 x}{\nu}\frac{\partial^3 f}{\partial \eta^3}. \qquad \text{Eq. 9}$$

While variable θ can be reduced to:

$$\frac{\partial T}{\partial t} = a\left(T_w - T_\infty\right)\frac{\partial \theta}{\partial \tau}, \; \frac{\partial T}{\partial x} = 0,$$

$$\frac{\partial T}{\partial y} = \sqrt{\frac{a}{\nu}}\left(T_w - T_\infty\right)\frac{\partial \theta}{\partial \eta}, \; \frac{\partial^2 T}{\partial y^2} = \frac{a}{\nu}\left(T_w - T_\infty\right)\frac{\partial^2 \theta}{\partial \eta^2}. \qquad \text{Eq. 10}$$

Equations 2 and 3 now take the following form:

$$\frac{\partial^3 f}{\partial \eta^3} - \left(\frac{\partial f}{\partial \eta}\right)^2 + f\,\frac{\partial^2 f}{\partial \eta^2} + 1 - \frac{\partial^2 f}{\partial \eta\, \partial \tau} = 0, \qquad \text{Eq. 11}$$

$$\left(1 + \frac{4}{3}Rd\right)\frac{\partial^2 \theta}{\partial \eta^2} + \Pr\, f\,\frac{\partial \theta}{\partial \eta} - \frac{\partial \theta}{\partial \tau} = 0, \qquad \text{Eq. 12}$$

where the boundary conditions in Eq. 6 now become:

$$f\left(0,\tau\right) = S, \; \frac{\partial f}{\partial \eta}\left(0,\tau\right) = \alpha + \delta\frac{\partial^2 f}{\partial \eta^2}, \; \theta\left(0,\tau\right) = 1 + \beta\frac{\partial \theta}{\partial \eta} \text{ at } \eta = 0,$$

$$\frac{\partial f}{\partial \eta}\left(\eta,\tau\right) \to 1, \; \theta\left(\eta,\tau\right) \to 0 \text{ as } \eta \to \infty. \qquad \text{Eq. 13}$$

To test the stability of the steady flow solution $f(\eta) = f_0(\eta)$ and $\theta(\eta) = \theta_0(\eta)$ satisfying the boundary value problems in Eq. 2 and 3 as suggested by Awaludin et al. (2016):

$$f\left(\eta,\tau\right) = f_0\left(\eta\right) + e^{-\gamma\tau}F\left(\eta\right),$$
$$\theta\left(\eta,\tau\right) = \theta_0\left(\eta\right) + e^{-\gamma\tau}G\left(\eta\right), \qquad \text{Eq. 14}$$

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where γ is an unknown eigenvalue, and $F(\eta)$ and $G(\eta)$ are the small relatives of $f_0(\eta)$ and $\theta_0(\eta)$. We differentiated Eq. 14 with respect to η and τ (Bakar, 2018; Merkin et al., 2022); inserting into Eq. 11 and 12, we find the linearized problem:

$$\frac{\partial^3 F}{\partial \eta^3} - \left(2 f_0' - \gamma\right)\frac{\partial F}{\partial \eta} + f_0 \frac{\partial^2 F}{\partial \eta^2} + f_0'' F - \frac{\partial^2 F}{\partial \eta \partial \tau} = 0, \qquad \text{Eq. 15}$$

$$\left(1 + \frac{4}{3} Rd\right)\frac{\partial^2 G}{\partial \eta^2} + \Pr f_0 \frac{\partial G}{\partial \eta} + \Pr F \theta_0' + \gamma \frac{\partial G}{\partial \eta} = 0, \qquad \text{Eq.16}$$

with the boundary conditions:

$$F(0,\tau) = 0, \quad \frac{\partial F}{\partial \eta}(0,\tau) = 0, \quad G(0,\tau) = 0,$$
$$\frac{\partial F}{\partial \eta}(\eta,\tau) \to 0, \quad G(\eta,\tau) \to 0 \quad \text{as } \eta \to \infty. \qquad \text{Eq. 17}$$

The functions $F = F_0(\eta)$ and $G = G_0(\eta)$ in Eq. 15 and 16 identify the initial growth or decay of the solution as in Eq. 14. Hence, the following unknown linear eigenvalues should be considered in order to solve the corresponding numerical methods, which are:

$$F_0''' - \left(2 f_0' - \gamma\right) F_0' + f_0 F_0'' + f_0'' F_0 = 0, \qquad \text{Eq. 18}$$
$$\left(1 + \frac{4}{3} Rd\right) G_0'' + \Pr\left(f_0 G_0' + \theta_0' F_0\right) + \gamma G_0' = 0, \qquad \text{Eq. 19}$$

subjected to boundary conditions:

$$F_0(0) = 0, \quad F_0'(0) = 0, \quad G_0(0) = 0,$$
$$F_0'(\eta) \to 0, \quad G_0(\eta) \to 0 \quad \text{as } \eta \to \infty. \qquad \text{Eq. 20}$$

The analysis of stability for the corresponding problems of $f_0(\eta)$ and $\theta_0(\eta)$ are determined by smallest eigenvalues γ for selected values of involving parameters such as radiation parameter *Rd*, Prandtl number Pr, suction parameter *S*, shrinking parameter α and slip parameter δ and β, corresponding to velocity and thermal slip, respectively.

3. RESULTS AND DISCUSSION

Marangoni convection boundary layer flow in the presence of thermal radiation with suction/injection effects are analyzed numerically by Mat et al. (2013). The effects of thermal radiation parameters and the suction/injection parameter on the temperature profiles were presented in graphical form and thoroughly examined. The governing equations were transformed into ordinary differential equations using appropriate transformations and were then solved numerically by the shooting method. Comparisons with Hamid et. al (2011a) are performed and the results are in excellent agreement. It was found that the solutions for the constant exponent or the similarity parameter $\beta < 0.5$ were non-unique (dual solution). Results show that the dual solution exists for a certain range of the governing parameters. It could be drawn from the present results that when the radiation parameter increased, the heat transfer rate at the surface decreased. It was also shown that the imposition of suction was to decrease the surface temperature gradient, whereas injection showed the opposite effects. The solution is unstable if the value of the smallest eigenvalue is negative, while it is stable if the smallest eigenvalue vice versa.

4. CONCLUSION

In order to determine which of these solutions are physically realisable in practice, we have derived the stability analysis theoretically. The stability of the flow can be tested by looking at the polarity of the smallest eigenvalue itself. A stability analysis has been set up numerically to show that the upper branch solutions are stable and physically realizable, while the lower branch solutions are not stable and, therefore, not physically possible.

Declaration of Interest

The authors declare that there is no conflict of interest.

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