

CORRELATES OF MATHEMATICS ATTITUDE AND PROBLEM SOLVING BEHAVIOUR AMONG TEHERAN, REPUBLIC OF IRAN YEAR 10 STUDENTS

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Abstrak

Kajian korelasi dijalankan untuk menyiasat hubungan antara sikap matematik dan tingkahlaku semasa penyelesaian masalah matematik. Subjek kajian ini terdiri daripada 150 pelajar Matematik dan Fizik di peringkat Gred 10 dari Teheran, Republik Iran. Tiga kawasan telah dipilih secara rawak daripada 22 kawasan di bandar Teheran, dan sepuluh pengetua dari 10 buah sekolah menengah atau tinggi telah memberikan persetujuan mereka untuk mengambil bahagian dalam kajian ini. Secara keseluruhan, 150 pelajar telah mengambil bahagian dalam sesi pengajaran dan pembelajaran dan mereka diminta memberi maklum balas kepada dua soal selidik iaitu, *Mathematical Problem Solving Behaviours Scale* (MPSB) and *Mathematics Attitude Scale* (MAS). Soal selidik MPSB (tingkahlaku semasa penyelesaian masalah matematik) terdiri daripada tiga sub-skala, iaitu proses ketelitian dan meta-kognitif, aspek produktif dan pendekatan teknikal, manakala MAS (sikap matematik) terdiri daripada enam sub-skala iaitu, aspek *affect*, kecekapan kognitif matematik, nilai matematik, kesukaran dalam matematik, minat terhadap matematik dan daya usaha dalam pembelajaran matematik. Hasil kajian menunjukkan bahawa terdapat hubungan positif dan sederhana yang signifikan di antara sikap matematik dengan tingkah laku semasa penyelesaian masalah matematik dalam kalangan pelajar Gred 10 di Teheran, Republik Iran.

Kata kunci *Pembelajaran Matematik, sikap matematik, tingkah laku semasa penyelesaian masalah Matematik, strategi penyelesaian masalah*

Abstract

A correlation study was conducted to investigate the association between mathematics attitudes and mathematical problem solving behaviour. The subjects of this study were 150 Mathematics and Physics grade 10 students from Teheran, Republic of Iran. Three regions were selected randomly from 22 regions of Teheran city, and ten high schools principals give their consent to participate in the study. Altogether, 150 students participated during school sessions and were asked to response to two namely, *Mathematical Problem Solving Behaviours Scale* (MPSB) and *Mathematics Attitude Scale* (MAS). Students' mathematical problem solving behaviour comprise

of three sub-scales, namely precise and meta-cognitive process, productive aspects and technical approach, whilst mathematics attitude comprise of affect, mathematics cognitive competence, mathematics value, difficulty in mathematics, interest in mathematics and effort in mathematics learning sub-scales. The results indicated that there is a high positive significant correlation between students' mathematics attitudes with students' mathematical problem solving behaviour.

Keywords *Mathematics learning, mathematics attitudes, problem solving behaviour, problem solving strategy*

INTRODUCTION

Problem solving, which has been known as the heart of mathematics, occupied a central place since antiquity (Stanic & Kilpatrick, 1988), should be considered as a central focus of mathematics curriculum (NCTM, 2000). The growth rate of careers related to the mathematical science and engineering work force is good indication of the importance of mathematics in today's world. Ramdass and Zimmerman (2008) also reiterate that problem solving is a basic skill needed by today's learners. Guided by recent research in problem solving, changing professional standards, new workplace demands, and recent changes in learning theory, educators are revising curricula to include integrated learning environments which encourage learners to use higher order thinking skills, and in particular, problem solving skills.

An activity of problem solving provides students opportunities to construct and experience the power of mathematics. It helps students to understand mathematical content and leads them in applying their knowledge in real world problems. Actually, mathematics is a way of thinking and requires a great deal of attention, particularly when multiple steps are involved in problem solving process (Amini, Alamolhodaei & Radmehr, 2012). More important than just having the knowledge of mathematical concepts, an individual must have the capacity to plan out the problem solving, monitor their progress and evaluate or check their worked solutions when they have completed problem solving (Wood, 2002). According to Ramdass and Zimmerman (2008), success in acquiring mathematical knowledge and problem solving skills is critical as both have direct relation to individuals' future college and job opportunities. The mathematical problem solving performance of Iranian students in the Third International Mathematics and Science Study (TIMSS) was way below the international average (TIMSS, 2003, 2009). However, only a few research studies and government-funded projects have been launched to investigate this matter. Consequently, the performance of students in Teheran, as a capital city of Iran, should be explored.

Globally students always lament on having to learn mathematics let alone learning mathematics with understanding (Eynde, Corte, & Verschaffel, 2006). This phenomenon applies to Iranian students (Gooya, 2003). What leads students to simply solve mathematical problems without thinking or they simply do not solve and think about mathematics problems? Convincing the students to solve mathematical problems is not easily accomplished unless educators have more knowledge of the important factor that may affect students' mathematics performance, like, attitudes

toward mathematics and problem solving behaviour. This recent social cognitive perspective in mathematics learning in which educators should take into account learners' characteristics has gain emphases in order to attain effective learning and problem solving in mathematics (Schunk, 2001).

Elia, Heuvel-Panhuizen, and Kolovou (2009) suggest that the ability to select an appropriate strategy in order to solve mathematics problems is an important consideration in teaching and learning mathematics. Hence, in order to gain better understanding about mathematical problem solving strategies, investigation into the different strategies among students' problem solving performance, specifically in Iran, is deemed necessary to facilitate improvement of problem solving performance in line with international standards such as TIMSS. It is clear that having information and knowledge about students' abilities, skills and performance, as the future work force is a critical strategy for leaders of the country (Ministry of Education, Iran, 2006). Hence, this study aimed to uncover the hidden aspects of mathematical learning, specifically, the relationship between psychological perspectives that are related to students' problem solving performance.

Research Objective

1. To determine the level of students' mathematics attitudes and its sub-constructs (affect, mathematics cognitive competence, mathematics value, difficulty in mathematics, interest in mathematics and effort in mathematics learning), among Teheran Year 10 students.
2. To determine the level of students' mathematical problem solving behaviour and its sub-constructs (precise and meta-cognitive process, productive aspects and technical approach) among Teheran Year 10 students.
3. To determine the relationship between students' mathematics attitudes and its sub-constructs (affect, mathematics cognitive competence, mathematics value, difficulty in mathematics, interest in mathematics and effort in mathematics learning), with students' mathematics problem solving behaviour and its sub-constructs (precise and meta-cognitive process, productive aspects and technical approach) among Teheran Year 10 students.

Theoretical and Conceptual Framework

Pertaining to mathematical problem solving, Artzt and Armour-Thomas (1997) reported that students' ability to understand, explore, analyze, plan, implement, verify, and observe and listen are factors which can affect problem solvers' performance. In addition, each of these behaviours can be categorized as predominantly cognitive or metacognitive, where the working memory can be technically distinct between cognition and metacognition. The concept of cognition is concerning the processes of problem solving, and metacognition is involved in choosing and planning what to do and monitoring and regulating what is being done during problem solving.

Schoenfeld (1985) in ‘Mathematical Problem Solving’ book offered a framework for analyzing how and why people are successful (or not) when they engage on problem solving. He considers four aspects in his investigation of mathematical problem solving behaviour, namely, knowledge of students (proposition and procedural knowledge of mathematics), heuristics (strategies and techniques for problem solving such as working backwards, or drawing figures), control (decisions about when and what resources and strategies to use), and beliefs (a mathematical “world view”). At the same time, successful problem solving in mathematics requires students to be able to select and use task-appropriate cognitive strategies for understanding, representing, and solving problems (Schoenfeld, 1985; Mayer, 1992). Numerous researchers (Hofer, 1999; Dahl, Bals, & Turi, 2005; Paulsen & Feldman, 2005) concluded that, students’ mathematical attitudes influence the strategies they use for learning; hence develop students’ confidence critical thinking and improving the problem solving methods. Three examples of a problem solving heuristic are presented in Table 1.

Table 1 Steps in problem solving

John Dewey (1933)	George Polya (1998)	Stephen Krulik and Jesse Rudnick (1980)
Confront Problem	Understanding the Problem	Read
Diagnose or Define Problem	Devising a Plan	Explore
Conjecture Consequences of Solutions	Carrying Out the Plan	Select a Strategy
Test Consequences	Looking Back	Solve
Test Consequences		Review and Extend

Source: Carson, 2007

Lester (1994) conducted a research to find the role of behaviour in problem solving processes and rejected the pure theory-based nature of mathematical solving problem process. He concluded that teaching strategies for solving problems is not adequate to improve students’ problem solving ability. In addition, more structured planning on instruction need to be considered for a long term period of teaching and learning in order to produce superior problem solvers. He stated that problem solving performance is a function of several independent factors such as knowledge, control, beliefs, and socio-cultural contexts. He defined “good” problem solvers as possessing more knowledge, well-connected knowledge, and rich schemata. Other studies had also explored the factors related to problem solving performance (Goldin & McClintock, 1984; Schoenfeld, 1985; Lester, 1994; Adibina & Putt, 1998; Trafton & Midgett, 2001; Yimer & Ellerton, 2002, Kadijevic, 2004; Carlson & Bloom, 2005; Downs & Downs, 2005; Debellis & Goldin, 2006).

In mathematics education, strategic behaviour is associated with higher problem-solving skills and achievement (Zimmerman & Martinez-Pons, 1988, 1990; Pape & Wang, 2003). According to these researchers, strategic learners use different motivational, cognitive, and metacognitive strategies to achieve their learning goals. Firstly, students will acquire mathematics facts, concepts, tools, and procedures

based on students' knowledge. Subsequently, they will focus on applied knowledge and conceptual understanding in the problem solving activity. Students will also go beyond and encompasses the unfamiliar situations, complex contexts, and multi-step problems in the solution of routine or non-routine problems. Many researchers state that the attitudes and beliefs about the nature of knowledge and learning are highly inter-related to motivational, cognitive, and meta-cognitive aspects of learning, thus enhancing students' mathematical problem solving ability (Braten & Stromson, 2005; Dahl, Bals & Turi, 2005; Paulsen & Feldman, 2005; Eynde, Corte & Verschaffel, 2006).

One notion about the meaning of attitudes is based on McLeod's (1992) study which is cited by Eleftherios and Theodosios (2007). McLeod states that attitude can be shown as positive and negative feeling. Besides this, negative and positive attitudes can be a consequence of replicated affects response to mathematics. Leder and Forgasz (2002), report that aspects such as teaching approaches, social and environmental factors, and educational system of countries are also among important items that affect mathematics problem solving performance. Hannula (2002) found that students' attitudes towards mathematics are associated to the superiority of teachers' teaching methods and the climate of the class. Study by Papanastasiou (2002) also indicated that school climate influences teaching approaches among mathematics teachers.

Maker (1982) noted, "There is a cognitive component in every affective objective and affective component in every cognitive objective." According to Schoenfeld (1985), four categories of knowledge/skills are needed for a learner to be successful in mathematics:

- a) Resources - proposition and procedural knowledge of mathematics,
- b) Heuristics - strategies and techniques for problem solving such as working backwards, or drawing figures,
- c) Control - decisions about when and what resources and strategies to use, and
- d) Beliefs - a mathematical "world view" that determines how someone approaches a problem.

Schoenfeld's theory is supported by extensive protocol analysis of students solving problems. The theoretical framework is based upon much other work in cognitive psychology. McLeod (1988) stated this relation with problem solving, "if the students obtain a solution to a problem, they typically express feelings of satisfaction, even joy," and these are important for problem solving performance. The National Council of Teachers of Mathematics (NCTM, 1989) and National Research Council (NRC, 1989) have encouraged mathematics educators to incorporate affective factors and cognitive factors such as algorithmic procedures and problem solving which defined by Schoenfeld (1985). The conceptual framework for this study is showed in Figure 1. This study investigates the relationship between mathematics attitudes and mathematical problem solving behaviour among Teheran Year 10 students based on the cognitive theory, attitude toward mathematics and theories of affect in mathematics education.

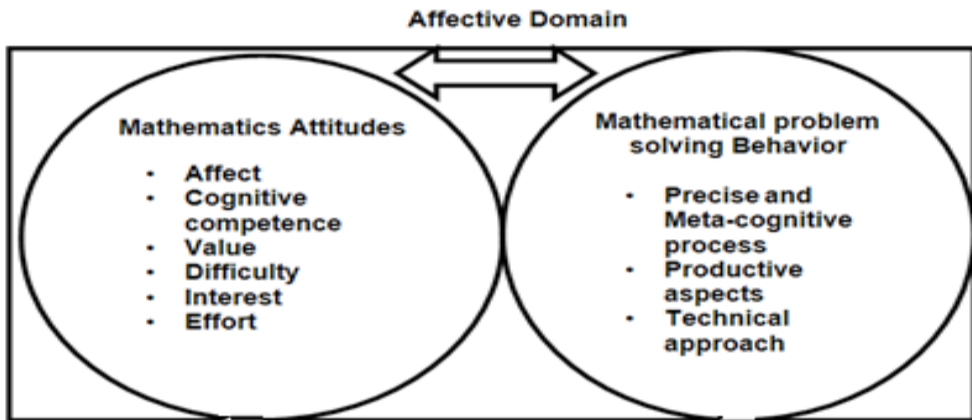


Figure 1 Conceptual Framework of Study

METHODOLOGY

A descriptive correlation design was used for the current study and two standard questionnaire, Students' Mathematics Attitudes and Mathematical Problem Solving Behaviour were used to gather information on the two constructs. Students' mathematics attitudes comprise of six sub-constructs namely affect, cognitive competency, value, difficulty, interest and effort. Mathematical problem solving behaviour consist of three sub-constructs namely precise and meta-cognitive process, productive aspects, and technical approach. The descriptive correlation research design is appropriate in investigating on psychological constructs where data can be used to explain the population understudy (Barribeau, Butler, Corney, Megan, Gault, & Gordon, 2005; Steiner, 2007).

The measured variables in this study were students' mathematics attitudes and mathematical problem solving behaviour. Mathematics Attitudes Scale (MAS) comprised of six constructs: affect, mathematics cognitive competence, mathematics value, difficulty in mathematics, interest in mathematics, and effort in mathematics learning. These sub-constructs of mathematics attitudes were consisted of 36 items from Schau (2003) and 7-point Likert rating scale (ranging from 1 as strongly disagree response rate to 7 as strongly agree response rate).

The next questionnaire used in this study was Mathematical Problem Solving Behaviour which consisted of three sub-constructs namely, precise and meta-cognitive process, productive aspects and technical approach, included 14 items from Perrent and Taconis (2009), were measured based on a 5-point Likert rating scale (ranging from 1 as strongly disagree response rate to 5 as strongly agree response rate).

The target population for this study was Teheran Mathematics and Physics Year 10 students. The total number of Mathematics and Physics year 10 students in Teheran are approximately 1500 students from 22 regions. Random sampling using the fish bowl techniques was used to select three regions, whereby, the number of each regions in Teheran (22 regions) was written on a piece of paper and was placed in a box, and

three numbers were selected. Three regions (3, 12, and 18) were selected randomly. Altogether there were 10 high schools, that included Mathematics and Physics year 10 students, with four boys' schools (112 students) and six girls' schools (168 students), and total 280 Mathematics and Physics Year 10 students.

Table 2 Distribution of sample of the study in Teheran (Regions 3, 12, 18)

Geographical Location	Gender	Student (10 th grade)	Number of School
Region 18	Male	60	2
	Female	82	3
Region 12	Male	27	1
	Female	26	1
Region 3	Male	25	1
	Female	60	2
Total		280	10

RESULTS

Altogether, 150 valid responses were collected in the study. The results for the mathematics attitude, mathematics problem solving behaviour, and the correlation between the variables were analyzed and discussed.

Students' Mathematics Attitude

The results (Table 3) indicated that the overall mathematics attitudes ranged from a minimum value of 2.97 to maximum value of 6.79, with a mean of 5.23 which indicates in general, students displayed high and positive mathematics attitudes. The rating scale is from 1 as strongly disagree response to 7 as strongly agree response. Five sub-constructs were assessed namely, affect, cognitive competence, value, difficulty, interest and effort with means of 5.21, 5.82, 5.02, 4.84, 5.28, and 5.19 respectively. The highest score among the sub-construct is of cognitive competence (mean= 5.82), indicating that most students strongly agreed to statements such as "they can learn mathematics" or "they can understand mathematics equations". Students also rated low on negatively worded items such as "they have no idea of what's going on in mathematics course" or "they make a lot of mathematics errors in mathematics learning"

Students' responses on the interest sub-construct indicated they are highly interested in learning, understanding, using, and communicating mathematics. The overall mean is 5.28 (SD = 1.30) indicating relatively high mathematics attitude on the sub-construct interest. In addition, based on the six items measuring the affect sub-construct, students agreed with statement "I will like mathematics," (with the highest mean value of 5.27) and disagreed with statement "I will get frustrated going over mathematics tests in class," (with the lowest mean value of 2.45), thus students' had positive perception about their affects towards mathematics.

Nine items were used to measure the value sub-construct, and students agreed with item “Mathematics skills will make me more employable” (with the highest mean value of 5.27) and disagreed with statement “Mathematics is not useful to the typical professional”, (with the lowest mean value of 2.13), thus students’ perception about attitudes toward usefulness, significance, and the importance of mathematics in their real life and career were positive.

On the other hand, based on seven items measuring difficulty sub-construct, students agreed with “Mathematics formulas are easy to understand,” (with the highest mean value of 5.41) and disagreed with statement “Learning mathematics requires a great deal of discipline” (with the lowest mean value of 2.74). Hence, it maybe concluded that students do not have much issue related to difficulty of mathematics learning.

Table 3 Items of mathematics attitudes’ sub-constructs

Mathematics Attitude		Mean	SD
Composite score of Mathematics attitude		5.23	.83
Affect			
3.	I will like mathematics.	5.27	1.44
4.*	I will feel insecure when I have to do mathematics problems.	2.58	1.53
15.*	I will get frustrated going over mathematics tests in class.	2.45	1.49
18.*	I will be under stress during mathematics class.	3.31	1.75
19.	I will enjoy taking mathematics courses.	5.00	1.26
28.*	I am scared by mathematics.	2.67	1.45
Total	Affect	5.21	1.14
Cognitive competence			
5.*	I will have trouble understanding mathematics because of how i think.	2.21	1.21
11.*	I will have no idea of whta’s going on in this mathematics course.	2.33	1.38
26.*	I will make a lot of math errors in mathematics.	2.27	1.18
31.	I can learn mathematics.	6.10	0.76
32.*	I will understand mathematics equations.	6.09	0.71
35.*	I will find it difficult to understand mathematical concepts.	2.45	1.36
Total	Cognitive competence	5.82	0.78
Value			
7.*	Mathematics is worthless.	2.19	1.28
9.	Mathematics should be a required part of my professional training.	4.85	1.73
10.	Mathematics skills will make me more employable.	5.27	1.38

13.*	Mathematics is not useful to the typical professional.	2.13	1.01
16.*	Mathematics thinking is not applicable in my life outside my job.	3.15	1.59
17.	I use mathematics in my everyday life.	3.33	1.23
21.*	Mathematics conclusions are rarely presented in everyday life.	2.93	1.50
25.*	I will have no application for mathematics in my profession.	3.20	1.46
33.*	Mathematics is irrelevant in my life.		
Total	Value	5.02	1.00
Difficulty			
6.	Mathematics formulas are easy to understand.	5.41	1.43
8.*	Mathematics is a complicated subject.	3.63	1.70
22.	Mathematics is a subject quickly learned by most people.	5.07	1.47
24.*	Learning mathematics requires a great deal of discipline.	2.74	1.46
30.*	Mathematics involves massive computation.	3.63	1.70
34.*	Mathematics is highly technical.	3.20	1.46
36.*	Most people have to learn a new way of thinking to do mathematics.	3.38	1.72
Total	Difficulty	4.84	1.10
Interest			
12.	I am interested in being able to communicate mathematical information to others.	4.70	2.04
20.	I am interested in using mathematics.	5.11	1.48
23.	I am interested in understanding mathematical informatin.	5.61	1.22
29.	I am interested in learning mathematics.	5.69	1.32
Total	Interest	5.28	1.30
Effort			
1.	I plan to complete all of my mathematics assignments.	5.11	1.48
2.	I plan to work hard in my mathematics course.	5.25	1.36
14.	I plan to study hard for every mathematics test.	5.21	1.54
27.	I plan to attend every mathematics class session.	5.20	1.44
Total	Effort	5.19	1.38

“*”: Reversed Items

Students' Mathematical Problem Solving Behaviour

Three sub-constructs were used to assess students' mathematical problem solving behaviour namely, precise and meta-cognitive process, productive aspects, and technical approach. A five-point Likert scale ratings were used and the overall ratings of students mathematics problem solving behaviour was positive or high with a mean of 3.40 (also with a minimum value of 1.69, maximum value of 5.00, see Table 4). Based on these results, students displayed a moderate level of agreement about precise and meta-cognitive processes in mathematical problem-solving which included, checking the answer before starting with the next problem (mean of 3.43), checking solution method (mean of 3.21), formulate in a precise way (mean of 3.19), regularly asking oneself whether he/she is on the right track (mean of 3.11), making estimation of the answers to assignment (mean of 2.77).

However, students' responses were of higher level of agreement in productive aspects of mathematical problem solving. Majority of students agreed with statement like "The solution of a mathematical assignment is sometimes much unexpected". Students displayed low level of agreement "Applied mathematics has available ready-made methods to solve mathematical problems from other technical disciplines" and "I solve mathematical assignments in one way only". Hence, students' perceived that production of mathematics solutions maybe unexpected and can be produced in specifically one way.

Table 4 also indicated that students displayed a moderate level in technical approach to mathematical problem solving which included reading precisely, sketch and drawing table, always precise answer, learning by heart not necessary, insight and work, and common sense (most students agreed with statement like "I should always give a very precise answer in a mathematical assignment: for example it is not allowed to round off the numbers in your answer").

Table 4 showed that from the five items measuring precise and meta-cognitive process sub-construct, students agreed with item "When I have found the answer of a mathematics assignment I check it, before I start with the next one" (with the highest mean value of 3.43). This indicated that students worked by reading/thinking/writing down the solution, then they look at the answer to see perform a direct control, whenever possible, students do check an answer. Students neither disagreed nor agreed with item "Sometimes I first try to estimate the outcome of a mathematics assignment" (with the lowest mean value of 2.77). This indicated that some of the students agreed to estimate the answer when they solve the problem.

From the three items measuring productive aspects sub-construct, students agreed with item "The solution of a mathematical assignment is sometimes much unexpected" (with the highest mean value of 3.55). It showed that nowadays a totally new argument could be needed for a solution; that is what makes mathematics exciting and students have often seen surprising answers. Students disagreed with item "Applied mathematics has available ready-made methods to solve mathematical problems from other technical disciplines" (with the lowest mean value of 1.85). This indicated that students agreed to recognize structural analogies of application methods versus tailoring standard methods.

From the six items measuring technical approach, according to Table 4, students strongly agreed with item “Mathematics is 90% insight and 10% work” (with the highest mean value of 3.35). It showed that students sometimes have to read the question again and again before they understand what is actually asked for. Insight is very important at school and it should be acquired enough. Students neither disagreed nor agreed with item “When I do not succeed in solving a mathematics problem, I often try to make a sketch, a drawing or a table” (with the lowest mean value of 3.09). It showed that the students gained insight into the problem and the solution by using the sketches or drawings.

Table 4 Mathematical problem solving behaviours’ constructs

Mathematical problem solving behaviour		Mean	SD
Total	Mathematical problem solving behaviour	3.40	0.72
Precise and meta-cognitive process			
3.	Whilst solving a mathematics problem I regularly ask myself whether I am on the right track.	3.11	1.12
4.	Sometimes I first try to estimate the outcome of a mathematics assignment.	2.77	1.05
5.	When I have found the answer of a mathematics assignment I check it, before I start with the next one	3.43	1.13
6.	Sometimes I try to investigate whether my own solution method of a mathematics assignment is the most convenient.	3.21	1.11
7.	In mathematics, it is often necessary to formulate in a precise way.	3.19	1.11
Total	Precise and meta-cognitive process	3.14	0.86
Productive aspects			
12.*	I solve most mathematical assignments in one way only.	2.02	1.20
16.	The solution of a mathematical assignment is sometimes much unexpected.	3.55	1.23
17.*	Applied mathematics has available ready-made methods to solve mathematical problems from other technical disciplines.	1.85	1.14
Total	Productive aspects	3.85	0.65
Technical approach			
1.	With a mathematical assignment I always start by very precisely reading through the givens and what is asked for.	3.21	1.08
2.	When I do not succeed in solving a mathematics problem, I often try to make a sketch, a drawing, or a table.	3.09	1.17
8.	I should always give a very precise answer in a mathematical assignment: for example it is not allowed to round off the numbers in your answer.	3.17	1.13
9.	In mathematics, it is hardly necessary to learn something by heart.	3.18	1.11

10.	Mathematics is 90% insight and 10% work.	3.35	1.11
11.	Some mathematics assignments can be solved by using common sense, without using rules.	3.21	1.07
Total	Technical approach	3.20	0.93

“*”: Reversed Items

Correlations between Students’ Mathematics Attitudes and Problem Solving Behaviour

The findings in Table 5 revealed that there is a moderate positive significant correlation between overall mathematics problem solving behaviour and mathematics attitudes ($r = .489, p < .01$). This findings suggest that the two variables are related and that mathematics attitudes may enhance learners mathematical problem solving behaviour in general or vice versa.

Specifically, as presented in Table 5, there is a moderate positive and significant correlation between affect sub-construct of mathematical attitudes and precise and metacognitive process as mathematical problem solving behaviour subscale ($r = .312, p < .05$) indicating that students’ affect during the mathematics learning is related to students’ meta-cognitive process during problem solving. There is a low positive significant correlation between affect and productive aspects subscale ($r = .269, p < .05$), indicating that students’ affect during the mathematics learning is related to students ability to use productive methods in solving the mathematics problems. There is a moderate positive significant correlation between affect and technical approach subscale ($r = .300, p < .05$) indicating that students’ mathematics affect is related to students’ ability to employ different techniques to solve mathematics problems.

There is a moderate positive significant correlation between cognitive competence with precise and meta-cognitive process ($r = .385, p < .05$). Findings suggest that students’ cognitive competence is moderately related to their meta-cognitive process during mathematics problem solving. However, there is a low positive significant correlation between cognitive competence with productive aspects ($r = .284, p < .05$), indicating that students’ cognitive competence is related to students’ mathematical productivity. There is a moderate positive significant correlation between cognitive competence with technical approach ($r = .367, p < .05$). This indicates that students’ mathematical competence is related to students’ ability to solve mathematics problems by different techniques.

With regards to value as mathematics attitudes sub-construct, there is a low positive significant correlation between value and precise and meta-cognitive process ($r = .234, p < .05$). This indicates that students’ values toward usefulness, significance, and the importance of mathematics in their real life and career is related to students’ meta-cognitive process during problem solving or in search for correct answers to mathematics problems.

There is a low positive significant correlation between value and productive aspects ($r = .176, p < .05$). Hence, students’ value toward usefulness, significance, and the importance of mathematics is related to students’ ability to solve mathematics

problems. There is a low positive significant correlation between value and technical approach ($r = .225, p < .05$), indicating that students' value toward usefulness, significance, and the importance of mathematics is related to students' ability to employ different techniques to solve mathematics problems.

Table 5 also indicated that there is a moderate positive significant correlation between interest (a sub-construct to mathematics attitudes) with precise and meta-cognitive process ($r = .326, p < .05$). This indicates that, several aspects of interest in mathematics is related to students' meta-cognitive process during mathematics problem solving activity. There is a low positive significant correlation between interest and productive aspects, ($r = .287, p < .05$). There is a moderate positive significant correlation between interest and technical approach ($r = .315, p < .05$). Hence, interest related to mathematics attitudes is correlated to students' ability to employ different techniques in solving mathematics problems.

In addition, results indicated that there is a moderate positive significant correlation between effort with precise and meta-cognitive process ($r = .476, p < .05$). Hence, students' effort to learn mathematics is related to students' meta-cognitive process during problem solving activity. There is a moderate positive significant difference between effort and productive aspects ($r = .328, p < .05$), so students' effort to learn mathematics is related to students' ability to solve mathematics problems. There is a moderate positive significant correlation between effort with technical approach ($r = .300, p < .05$). Hence, students' effort to learn mathematics is related to students' ability to employ different techniques to solve mathematics problems.

Table 5 Correlation between mathematics attitudes with mathematics problem solving behaviour

Item Description	1	2	3	4	5	6	7	8	9	10
1 Attitudes	1									
2 Affect	.821**	1								
3 Cognitive competence	.733**	.741**	1							
4 Value	.753**	.520**	.405**	1						
5 Difficulty	.769**	.765**	.653**	.449**	1					
6 Interest	.816**	.548**	.431**	.707**	.459**	1				
7 Effort	.591**	.219**	.251**	.310**	.224**	.428**	1			
8 Problem solving behaviour	.489**	.333**	.395**	.242**	.340**	.350**	.492**	1		
9 Precise and meta-cognitive	.473**	.312**	.385**	.234**	.323**	.326**	.496**	.940**	1	
10 Productive aspects	.332**	.269**	.284**	.176*	.230**	.287**	.235**	.739**	.526**	1
11 Technical approach	.473**	.300**	.367**	.225**	.333**	.315**	.525**	.947**	.901**	.538**

“*”: Significant at 0.05 level; “***”: Significant at 0.01 level

DISCUSSION AND CONCLUSION

Relationship between students' mathematics attitudes and mathematical problem solving behaviour were investigated in this study in which positive correlations was found between the two variables. Moreover, the positive significant correlation was found between six sub-constructs of students' mathematics attitudes and three sub-constructs of mathematical problem solving behaviour. Several studies in mathematics education provide strong evidence for the association between students' mathematical problem solving behaviour and mathematics attitudes construct (Hofer 1999; Mason, 2003; Mason & Scrivani, 2004; House 2006; Koller, 2007). Many scholars (Kloosterman, 1988; Kloosterman & Cougan, 1994; Kloosterman et al.; 1996; Op't Eynde, et al., 2000; Mason, 2003) found that students' affects (beliefs about importance of mathematics and beliefs about ones' ability in mathematics) is related to mathematics problem solving ability. Similarly, the findings of the present investigation indicated that students' mathematics attitudes and its sub constructs were vigorously influenced by their mathematics problem solving behaviour.

This indicate that students' mathematics attitudes encompassing intellectual knowledge and skills, usefulness, relevance, and worth of mathematics in personal and professional life, difficulty of mathematics as a subject, individual interest in mathematics, and amount of work the student expends during mathematics learning is related to students' meta-cognitive process during mathematics problem solving activity and technical aspects of problem solving encompassing creative method to solve mathematics problems and different techniques employed to solve mathematics problem.

The findings of this research have generated recommendations for future research about the role of students' attitudes toward mathematics and mathematical problem solving behaviour in context of high school students. It is noted in the literature review (McLeod, 1992; Hannula, 2002; Eleftherios & Theodosios, 2007; Chen, 2002; Fardin, Alamolhodaei, & Radmehr, 2011; Artzt & Armour-Thomas, 1997; Schau, 2003; Muir, Beswick, & Williamson, 2008), that the most significant and informative studies incorporated data from several sources and acknowledged the association between individual and performance in mathematics problem solving. It seemed that a more detailed investigation into the interaction between these variables had the possibility of revealing some interesting information about influences of students' mathematics attitudes and problem solving behaviour.

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