MATHEMATICAL PROBLEM-SOLVING BEHAVIOR OF SUCCESSFUL PROBLEM SOLVERS

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Abstract

The purpose of this study was to investigate the mathematical problem-solving behavior of mathematics teachers during their problem-solving endeavors and how their mathematical problem-solving behavior ensured their problem-solving success. Specifically for this study, the teachers' mathematical problem-solving behavior involved their problem-solving strategies, decisions, and beliefs about mathematics. In view of this, the researcher employed the generic qualitative study design and the collected data included the teacher' interviews and observations, and their completed mathematics problems document. The teachers' data were then analyzed based on the conceptual framework of this study which is the conceptual framework of mathematical problem-solving behavior. Also, the mathematics teachers selected for this study were based on their level of degree qualification, years of experience in teaching mathematics, and the education level that they have taught. The findings of the study revealed in detail the teachers' mathematical problem-solving behaviors however similar or different they were altogether successful in acquiring the solutions to the tasks given. Hence, this study concluded with the mathematical problem-solving behavior of successful problem solvers. This presentation portrayed successful problem solver's mathematical problem-solving behavior which consisted of solution strategies, decision-making episodes, and beliefs about nature and doing mathematics. Thus, this presentation best explain what it takes to be a successful problem solver which involve an individual mathematical problem-solving behavior that consist of her/his problem-solving strategies, decision-making episodes, and beliefs about nature of mathematics and doing mathematics.

Keywords *Mathematical problem-solving behavior, Problem-solving strategies, Decision-making episodes, Beliefs about nature of mathematics and beliefs about doing mathematics*.

Abstrak

Kajian ini bertujuan untuk mengkaji "tingkah laku" guru-guru matematik dalam proses penyelesaian masalah matematik ("mathematical problem-solving behavior of mathematics teachers") dan bagaimana "mathematical problem-solving behavior" guru tersebut dapat menentukan bahawa mereka sebagai penyelesai masalah matematik yang berjaya. "Mathematical problem-solving behavior" guru-guru ini berfokuskan kepada strategi penyelesaian masalah, membuat keputusan dan kepercayaan mengenai matematik. Sehubungan itu, pengkaji menggunakan kaedah kualitatif yang bersifat generik dan data dikutip melalui temubual, pemerhatian dan

dokumen tentang masalah matematik yang telah diselesaikan oleh guru. Data-data tersebut dianalisis berdasarkan kepada kerangka konseptual kajian ini iaitu kerangka konseptual "mathematical problem-solving behavior". Guru yang terlibat dalam kajian ini dipilih berdasarkan kelulusan akademik mereka, tahun pengalaman mengajar matematik dan peringkat pendidikan yang telah mereka mengajar. Dapatan kajian menunjukkan bahawa walaupun terdapat persamaan atau perbezaan di antara "mathematical behavior" guru-guru tersebut, mereka didapati berjaya menyelesaikan masalah matematik yang diberi. Oleh itu kesimpulan dapat dibuat daripada dapatan kajian ini iaitu dengan memperlihatkan "mathematical problem-solving behavior" penyelesai masalah matematik yang berjaya. Ia menunjukkan bahawa "mathematical problem-solving behavior" penyelesai masalah matematik yang berjaya merangkumi strategi penyelesaian masalah, episod membuat keputusan serta kepercayaan mengenai sifat matematik dan membuat matematik. Justeru, apa yang diperlukan oleh seseorang individu itu supaya menjadi penyelesai masalah matematik yang berjaya adalah dari segi "mathematical problem-solving behavior" seseorang itu yang merangkumi strategi penyelesaian masalah, episod membuat keputusan serta kepercayaan mengenai sifat matematik dan membuat matematik.

Kata kunci *"Tingkah laku" matematik, Strategi penyelesaian masalah, Episod membuat keputusan, Kepercayaan mengenai sifat matematik dan membuat matematik*

INTRODUCTION

The quality of any education system cannot exceed the quality of its teachers (Barber & Mourshed, 2007; Mourshed, Chijioke, & Barber, 2010). Moreover, a recurring positive relationship exists between the teachers' teaching and their students' learning and consequently, the teachers' mathematical problem-solving behaviors are closely associated with their students' mathematical problem-solving behaviors (Ball & McDiarmid, 1990; Bransford et al., 2005; Calderhead, 1993; Clark & Lampert, 1986; Clark & Peterson, 1986; Darling-Hammond, 1999; Floden & Klinzing, 1990; Hill et al., 2005; Sabri Ahmad et al., 2006; Thompson, 1992). Hence, the teachers' mathematical problem-solving behaviors which guided, influenced, and ensured their approaches and success during their solution attempts unravel that they have misconceptions and gaps similar to those of their students (Ball & McDiarmid, 1990).

Malaysian mathematics teachers were found giving less emphasis: (i) in representing relationships through writing equations and interpreting data in tables or graphs; (ii) in reasoning and working on problems for which there was no immediate and obvious method of solution; and (iii) in relating what is being learned in mathematics to the daily-live situations during their problem-solving activities (Kementerian Pendidikan Malaysia, 2000; Mullis et al., 2000; Mullis et al., 2004; Mullis et al., 2007). It follows that only a minority of their students exhibited solving and explaining solution strategies in non-routine and real-life problem-solving situations (Kementerian Pendidikan Malaysia, 2000; Mullis et al., 2000; Mullis et al., 2004; Mullis et al., 2007). Also, only a minority of their students exhibited organizing and interpreting information from problem statements in equations or tables or graphs (Kementerian Pendidikan Malaysia, 2000; Mullis et al., 2000; Mullis et al., 2004; Mullis et al., 2007).

Little is known regarding the mathematical behavior of mathematics teachers particularly during their problem-solving endeavors that is, there is still lack of resources and valuable and important information concerning the mathematical problem-solving behavior of mathematics teachers (Floden & Klinzing, 1990). Moreover, there is lack of clear understandings, and what and the know-how among mathematics teachers when they do mathematics (Clark & Lampert, 1986; Floden & Klinzing, 1990; Hill et al., 2005; Nik Azis Nik Pa, 2008). Importantly, this information are about the behavior that influence and guide the teachers' practices in mathematics and which relate to how far is their mathematics teaching effectiveness (Bransford et al., 2005; Calderhead, 1993; Clark & Peterson, 1986; Floden & Klinzing, 1990; Hill et al., 2005; Ryans, 1963; Sabri Ahmad et al., 2006; Thompson, 1992). This information can then provide understandings and insights about the problems faced during their practices in mathematics and correspondingly, suggestions for solutions in improving their practices in mathematics and to become successful problem solvers (Calderhead, 1993; Fennema & Franke, 1992; Hill et al., 2005; Sabri Ahmad et al., 2006).

Thus, the focus of this study was to investigate the mathematical problem-solving behavior of mathematics teachers during their problem-solving endeavors and how this behavior ensured their problem-solving success. The mathematical problem-solving behavior concerned consisted of their problem-solving strategies, their decisions, and their beliefs about mathematics. It follows that the research questions for this study are:

Research Question 1: What problem-solving strategies did the mathematics teachers use when they were solving the mathematics problems?

Research Question 2: (a) What were the mathematics teachers' decisions that guided their work when they were solving the mathematics problems? (b) How did the mathematics teachers' decisions guide their work when they were solving the mathematics problems?

Research Question 3: (a) What were the mathematics teachers' beliefs about mathematics that influenced their approaches when they were solving the mathematics problems? (b) How did the mathematics teachers' beliefs about mathematics influence their approaches when they were solving the mathematics problems?

Research Question 4: How did the mathematics teachers' mathematical problem-solving behavior ensure their problem-solving success?

METHODOLOGY

The first research question was to investigate the utilized problem-solving strategies when the teacher-participants of the study were solving the mathematics problems assigned in the study. The following research question was to unravel the decisions specifically the decision-making episodes that the teachers went through during their solution processes and consequently, how these episodes guided their solution processes with the given mathematical tasks. To unravel the teachers' beliefs about mathematics

particularly their beliefs about nature of mathematics and doing mathematics was the third research question. It also included how these beliefs influenced their working processes with the mathematics problems. The last research question investigated and concluded how the teachers' mathematical problem-solving behavior which consist of their strategies, decision-making episodes, and beliefs about nature of mathematics and doing mathematics ensured their problem-solving success with the mathematics problems.

In view of this, the conceptual framework of this study adopted Silver's (1987) information processing system and specifically for this study, the three major components of the system are problem environment, long-term memory, and working memory. The framework of this study also adopted Schoenfeld's (1985, 1992) theoretical framework of mathematical behavior and for this study particularly, the components of the mathematical behavior consist of problem-solving strategies, decisions, and beliefs about mathematics. Moreover, researches that investigated mathematical problem-solving behavior of individuals employed a qualitative method in the collection of data and in the analysis of data (Borko & Putnam, 1996; Clark & Lampert, 1986; Floden & Klinzing, 1990; Hunting, 1997; Schoenfeld, 1989; Schoenfeld et al., 2000; Yinger, 1986; Zazkis & Hazzan, 1999; Zimmerlin & Nelson, 2000). Thus, the researcher employed the basic or generic qualitative study design for the present study (Merriam, 2001). This study design has four characteristics which are: (1) the researcher as the primary instrument of the data collection and data analysis, and the use of fieldwork in the study, (2) an inductive orientation to the analysis of data and the findings of the study that are richly descriptive, (3) it includes description, interpretation, and understanding of the processes involve in the study, and (4) it identifies the recurrent patterns in the form of categories and the delineation of the processes involve in the study.

The design was employed so that the researcher could gain an in-depth understanding of the mathematical behavior which the mathematics teachers portrayed during their problem-solving attempts. The teachers' mathematical behavior focused on their solution strategies, their decision-making episodes, their beliefs about nature of mathematics and doing mathematics, and consequently, how their mathematical behavior ensured their problem-solving success. The three mathematics teachers chosen for this study were based on their years of mathematics teaching experience, at what education level they were teaching, and their level of degree qualification. They were Form Four mathematics teachers with more than 10 years of mathematics teaching experience and masters graduates in mathematics education. For this study, the researcher addressed the three teachers-participants as Liyana, Choon Ling, and Devani.

Qualitative measures utilized in this study include the teacher-participants' interviews and observations, and their completed mathematics problems document. The mathematics problems are related to real-life situations and they were about temperature readings, heights of four sisters, elevator, office buildings, magazine subscriptions, and phone plans. These mathematics problems are mainly in the data representation and data analysis content domain with similar level of difficulty (Beaton et al., 1996; Mullis et al., 2000; Mullis et al., 2004). These problems additionally emphasized the application and making reasoning of the classroom-taught mathematics to interpret and solve real-world problems such as that relate to data representation and data analysis of real-world mathematical problems (Ministry of Education, 2004, 2004). The researcher retrieved the data about the teachers' mathematical behavior through

doing clinical interviews with the teachers when they were solving the mathematics problems and they were observed while doing so. These processes were audio-taped and video-taped all throughout. Upon completion, the completed mathematics problems documents were collected from the teachers when they have solved the assigned tasks. The collected data were analyzed primarily based on the conceptual framework of this study which is the conceptual framework of mathematical problem-solving behavior (Schoenfeld, 1985, 1992; Silver, 1987).

The researcher then employed Strauss and Corbin's (1990, 1998) two stages of coding which were the open coding stage and the axial coding stage in the analysis of the data. In the open coding stage, the data was taken apart, examined, compared, conceptualized, and categorized. In the axial coding stage, the data are put back together by making connections between categories and its specific features which were the subcategories, and these are validated against the data. The data of the study included the transcribed interview data, the summary of the observation field notes data, and the completed mathematics problems documents data of the teachers. In addition, the guideline of solutions to mathematics problems was utilized by the researcher to further assist in her analysis of the document data regarding the teachers' problem-solving success.

Thus, the findings of the study gave detailed explanation of the problem-solving strategies that the mathematics teachers used when they were solving the mathematics problems. The findings of the study also gave detailed explanation what decision-making episodes that the teachers went through and how these episodes guided them in their solution attempts with the problems. Additionally, the research findings revealed in detail the teachers' beliefs about nature of mathematics and doing mathematics and how these beliefs influenced them in their solution endeavors. Finally, the research findings revealed in detail how the mathematics teachers' mathematical problem-solving behavior ensured their problem-solving success.

FINDINGS OF THE STUDY

All the teachers, Liyana, Choon Ling, and Devani were found successful in acquiring correct solutions in their problem-solving attempts with the given mathematics problem and consequently, the teachers' mathematical problem-solving behavior contributed to their problem-solving success. Thus, this concludes for the last research question which is how did the mathematics teachers' mathematical problem-solving behavior affect their problem-solving success. The teachers' mathematical problem-solving behavior which consist of their problem-solving strategies, decisions (decision-making episodes), and beliefs about mathematics (beliefs about nature of mathematics and beliefs about doing mathematics) contributed to their problem-solving success.

It was discovered that Liyana had more **problem-solving strategies** than Choon Ling and Devani in obtaining correct answers during her solution attempts with the mathematics problems which were about temperature readings, heights of four sisters, elevator, office building, cheaper magazine subscription, and phone plans respectively. Liyana utilized 13 strategies altogether compared to Choon Ling (9 strategies) and Devani (12 strategies). Choon Ling and Devani also obtained correct answers during their solution attempts with the problems even though they utilized less strategies than Liyana. Basically, all the teachers used strategies about: (1) making analogy, (2) counting and doing arithmetic, (3) making lists, (4) making logical reasoning, (5) expressing in mathematical symbols, (6) verification by modifying

parts of the solution, (7) verification by using a different solution method, (8) making visualization, and (9) working forward, during their solution endeavors. Each strategy when utilized by the teachers played as an affirmative role in ensuring the teachers' problem-solving success.

The researcher found out that only Devani utilized the strategy about patterns. She was the only one among the three teachers that repeatedly mentioned and discussed about patterns being in mathematics and real-life situations during her problem-solving endeavors. The researcher also found out that only Liyana used making table strategy during her solution attempts. She did this so that she can understand and see clearly her solution plans and consequently find the solution to the assigned problem. Liyana too worked backward when faced with the assigned task. She recognized that in order to achieve the correct solution to the assigned task, she needed to use the working backwards strategy. Choon Ling and Devani did not show using these two strategies. They showed not requiring these two strategies for their solution acquisitions.

It was discovered that only Liyana and Devani made use of varying the problem strategy in their solution processes. In using this strategy both teachers were able to make corrections and were more certain of the correctness of their provided answers to the questions in the problems. Additionally, both teachers similarly utilized the strategy about verification by reworking the solution to the problem during their solution endeavors. They felt that to rework their solutions when dealing with the tasks given would give them confidence that they obtained the required correct solutions. Choon Ling did not show using these two strategies as she did not need to do so in order to obtain the correct solutions to the assigned tasks.

The mathematics teachers' problem-solving strategies can be summarized as in the following Table 1:

Table 1 Liyana, Choon Ling, and Devani: Problem-solving Strategies

Problem-solving Strategies		
Liyana	**Choon Ling**	**Devani**
Making analogy strategy	Making analogy strategy	Making analogy strategy
Counting and doing arithmetic strategy	Counting and doing arithmetic strategy	Counting and doing arithmetic strategy
Making lists strategy	Making lists strategy	Making lists strategy
Making logical reasoning strategy	Making logical reasoning strategy	Making logical reasoning strategy
Expressing in mathematical symbols strategy	Expressing in mathematical symbols strategy	Expressing in mathematical symbols strategy
-	-	Searching for patterns strategy
Making table strategy	-	-
Varying the problem strategy	-	Varying the problem strategy

Verification by reworking solution to problem strategy	-	Verification by reworking solution to problem strategy
Verification by modifying parts of the solution strategy	Verification by modifying parts of the solution strategy	Verification by modifying parts of the solution strategy
Verification by using a different solution method strategy	Verification by using a different solution method strategy	Verification by using a different solution method strategy
Making visualization strategy	Making visualization strategy	Making visualization strategy
Working backwards strategy	-	-
Working forward strategy	Working forward strategy	Working forward strategy
Total: 13 strategies	**Total: Nine strategies**	**Total: 12 strategies**

Basically, Liyana, Choon Ling, and Devani went through similar **decision-making episodes** that gave them guidance in their solution endeavors with the given tasks: reading episode, analysis episode, planning-implementation episode, and verification episode. These episodes are the main episodes in any of the teachers' solution attempts with the assigned mathematics problems and that seemed to guarantee their success in obtaining the correct solutions to the problems.

However, the researcher discovered only Devani went through the exploration episode when dealing with the given tasks. The researcher then discovered that Devani came up with interesting and unique possible solutions to the tasks given to her when she went through the exploration episode. This is somehow connected to the strategy that she utilized which was varying the problem strategy already discussed as above. By going through this episode, Devani explore various possibilities in tackling the tasks given and consequently, she came up with many possible solutions to the given tasks which were unique and interesting.

It was found also that Liyana and Devani went through the new information episode unlike Choon Ling. This finding corresponded to the finding concerning the usage of varying problem strategy by both teachers, Liyana and Devani. By utilizing this strategy, it would also indicate that the two of them went through the new information episode. When Liyana and Devani went through this new information episode, it guided them to later realize and discovered the correct information implied or given in the problem that they did not see at the beginning of their solution processes.

The following Table 2 below summarized the mathematics teachers' decisions (decision-making episodes) that guided their solution attempts with the given mathematics problems:

Table 2 Liyana, Choon Ling, and Devani: Decisions (Decision-making Episodes)

Decision-making Episodes		
Liyana	**Choon Ling**	**Devani**
Reading episode	Reading episode	Reading episode
Analysis episode	Analysis episode	Analysis episode
-	-	Exploration episode
New information episode	-	New information episode
Planning-implementation episode	Planning-implementation episode	Planning-implementation episode
Verification episode	Verification episode	Verification episode
Total: Five episodes	**Total: Four episodes**	**Total: Six episodes**

In general, all the teachers illustrated similar **beliefs about mathematics (beliefs about nature of mathematics and beliefs about doing mathematics)** that influenced their process of acquisition of solutions with the given mathematics problems. They are: beliefs about nature of mathematics is that mathematics and other subject matter areas are interrelated, mathematics is connected to the real world, mathematics is fun, and mathematics is interesting. They are also: beliefs about doing mathematics is that doing mathematics involves real-life situations, doing mathematics is fun, solutions to mathematics problems are pursued persistently, there is more than one correct method for solving a mathematics problem, and doing mathematics involves understanding. By having these beliefs in them, it indirectly and directly influenced the teachers in their solution attempts. Further, the above-mentioned beliefs that the teachers illustrated have guaranteed their success in attaining the problem goals.

Only Liyana exhibited believing about patterns which was her belief about nature of mathematics was that mathematics as a study of patterns and making sense of patterns. Also, her belief about doing mathematics was to look for patterns when solving the mathematics problem. This corresponded to the searching for pattern strategy that she utilized in her problem-solving endeavors as discussed above. She illustrated clearly in her problem-solving endeavors the existence of patterns and she made emphasis about this through her explanation for her answer to the question in the given problem. Unlike Choon Ling and Devani, they were not obvious believing about patterns during their solution attempts.

The mathematics teachers' beliefs about mathematics: beliefs about nature of mathematics and beliefs about doing mathematics are summarized as in the following table below. To begin is Liyana, Choon Ling, and Devani's beliefs about mathematics: beliefs about nature of mathematics:

Table 3 *Liyana, Choon Ling, and Devani: Beliefs About Mathematics (Beliefs About Nature of Mathematics and Beliefs About Doing Mathematics)*

Beliefs About Mathematics		
Liyana	**Choon Ling**	**Devani**
Beliefs About Nature of Mathematics	Beliefs About Nature of Mathematics	Beliefs About Nature of Mathematics
1. Mathematics and other subject matter areas are interrelated.	1. Mathematics and other subject matter areas are interrelated.	1. Mathematics and other subject matter areas are interrelated.
2. –	2. –	2. Mathematics as a study of patterns and making sense of the patterns.
3. Mathematics is connected to the real world.	3. Mathematics is connected to the real world.	3. Mathematics is connected to the real world.
4. Mathematics is fun.	4. Mathematics is fun.	4. Mathematics is fun.
5. Mathematics is interesting.	5. Mathematics is interesting.	5. Mathematics is interesting.
Total: 4	**Total: 4**	**Total: 5**
Beliefs About Doing Mathematics	Beliefs About Doing Mathematics	Beliefs About Doing Mathematics
1. Doing mathematics involves real-life situations.	1. Doing mathematics involves real-life situations.	1. Doing mathematics involves real-life situations.
2. Doing mathematics is fun.	2. Doing mathematics is fun.	2. Doing mathematics is fun.
3. –	3. –	3. Looking for patterns when solving mathematics problems.
4. Solutions to mathematics problems are pursued persistently.	4. Solutions to mathematics problems are pursued persistently.	4. Solutions to mathematics problems are pursued persistently.
5. There is more than one correct method for solving a mathematics problem.	5. There is more than one correct method for solving a mathematics problem.	5. There is more than one correct method for solving a mathematics problem.
6. Doing mathematics involves understanding.	6. Doing mathematics involves understanding.	6. Doing mathematics involves understanding.
Total: 5	**Total: 5**	**Total: 6**

The combination of all the three main components of the teachers' mathematical behavior, their problem-solving strategies, their decision-making episodes, and their beliefs about nature of mathematics and doing mathematics, whether there were similarities or differences, ensured the teachers' success in attaining the solution goals to the given mathematics problems.

Mathematical Problem-solving Behavior of Successful Problem Solvers

It follows that the mathematical problem-solving behavior of successful problem solvers include the components of problem-solving strategies, the decision-making episodes, and the beliefs about nature of mathematics and doing mathematics. The strategies were utilized to obtain the required correct solutions to the given mathematical tasks. The decision-making episodes that occurred during the teachers' solution attempts guided them thoroughly through their dealings with the mathematical tasks. Their beliefs about nature of mathematics and doing mathematics greatly influenced the teachers in the handlings of the given mathematical tasks. The mathematical tasks involved in the present study were real-life situational problems about temperature readings, heights of four sisters, elevator, office buildings, magazine subscriptions, and phone plans respectively.

Even though, there were similarities and differences in the teachers' mathematical behavior, they were altogether successful in attaining the solutions to the tasks given. When faced with the real-life situational problems, the obvious differences in the teachers' mathematical behavior were in the utilization of searching for pattern strategy and correspondingly, believing that mathematics as a study of patterns and making sense of the patterns and also looking for patterns. Also, by going through the exploration episode was the other obvious difference in the teachers' mathematical behavior that was discovered in the present study.

Clearly, the similarities among the successful problem solvers concerning their decision-making episodes would be when they went through the episodes of reading, analyzing, planning and implementing, and also verifying. Thus, the verifying part is connected to their common utilized strategy concerning verification. They too had similar beliefs about nature of mathematics and doing mathematics such as, they believed that mathematics is connected to the real world and in correspond to that, when they did mathematics it involved real-life situations. These beliefs concur with their common utilized strategies which were making analogies and visualizations and also logical reasoning.

In conclusion, the mathematical behavior of successful problem solvers which consisted of their problem-solving strategies, their decision-making episodes, their beliefs about nature of mathematics and doing mathematics ensured their success in their problem-solving endeavors. Consequently, the Mathematical Problem-solving Behavior of Successful Problem Solvers, is summarized and shown in detail as the following figure:

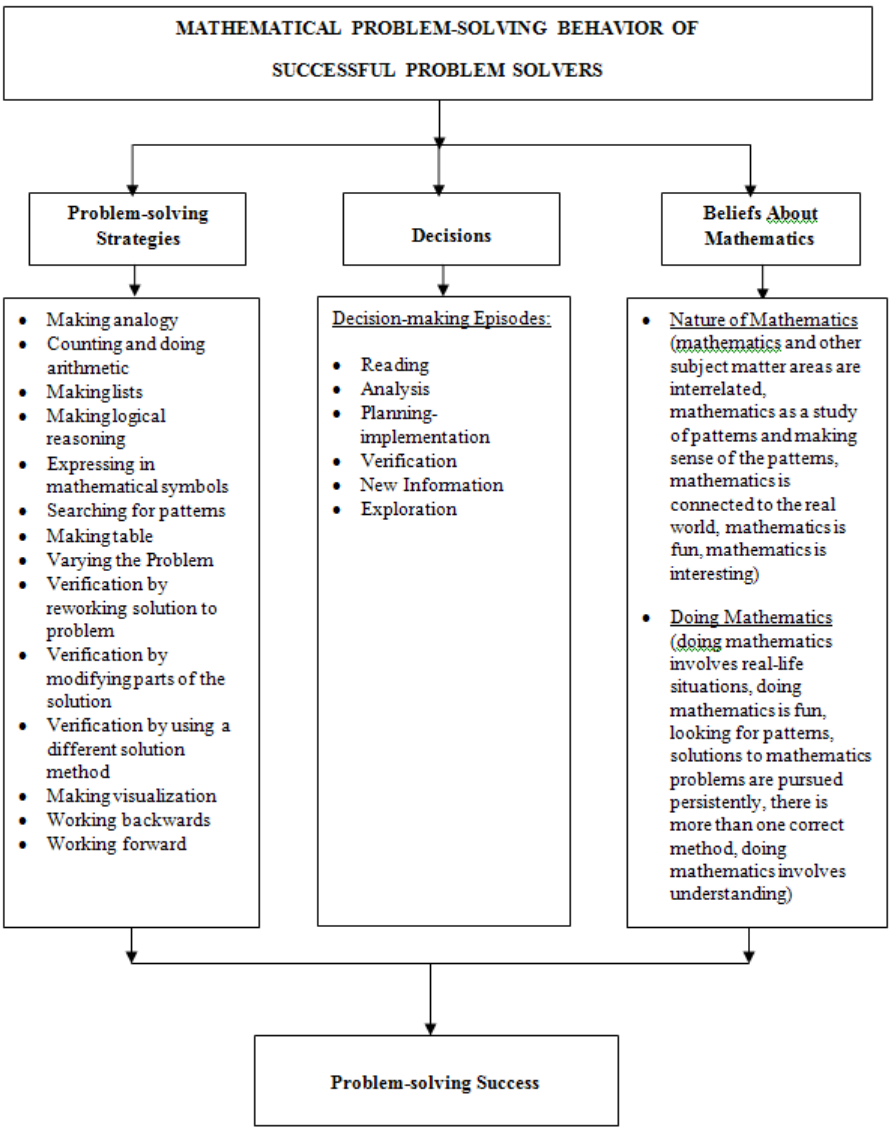

Figure 1: Mathematical Problem-solving Behavior of Successful Problem Solvers

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