ISSN 2289-7070

Assessing First Year Pre-service Teachers' Geometric Reasoning Ability on Two-Dimensional Shapes

Menilai Keupayaan Penaakulan Geometri Guru Pra-Perkhidmatan Tahun Pertama tentang Bentuk Dua-Dimensi

Chong Chin Yen[1] & Nor’ain Mohd Tajudin[2]
Mathematics Department, Faculty of Science and Mathematics,
Universiti Pendidikan Sultan Idris, Tanjong Malim, Perak
e-mail: [1]jengyin85@gmail.com, [2]norain@fsmt.upsi.edu.my

Abstract

Geometry serves both as an essential instructional tool in mathematics and a visualizing medium for regulating theoretical and real world constructs. Every mathematics teacher must be adequately equipped with the ways of understanding and thinking of the subject matter required to teach geometry. Relevantly, development of the underlying geometrical reasoning is often subject to the overarching concept of two-dimensional geometric shapes. The purpose of this study is to examine the first-year pre-service undergraduates’ reasoning ability about triangles and quadrilaterals. 140 second-semester undergraduates in lecture setting were given a standardized geometry test, namely the van Hiele Geometric Test (VHGT) which contains 20 multiple-choice questions. A descriptive analysis of the dichotomous data was conducted using SPSS and Winstep programs. The findings suggested that among those pre-service teachers, (a) at least 7.1% performed under the basic, visualization level, (b) at most 5.0% attained the required abstraction level, and (c) the rest belongs to group whose analysis level ability remained largely undifferentiated from the aspect of criterion used. This study indicated that reconstruction and reevaluation of textbook content may foster awareness towards rethinking about teachers’ own mathematical dispositions. By collectively or individually challenging their old belief structure, it is hoped that they could realize the epistemic value of having the shapes and its properties be related to each other as well as gaining ownership of such reconstructions of the concepts.

Key words geometry, van Hiele geometric level, reasoning ability, instructional tool, two-dimensional geometric shapes

Abstrak

Geometri merupakan alat penting dalam pengajaran matematik dan medium visualisasi untuk mengawal selia konstruk teori dan dunia sebenar. Setiap guru matematik mesti dilengkapi secukupnya dengan cara pemahaman dan pemikiran bagi suatu perkara yang diperlukan untuk mengajar geometri. Sama penting, pembangunan pemikiran geometri sering tertakluk kepada konsep yang bersifat menyeluruh bagi bentuk geometri dua-dimensi. Tujuan kajian ini adalah untuk mengkaji keupayaan penaakulan bentuk geometri segi tiga dan empat sisi dalam kalangan guru pra-perkhidmatan tahun pertama. 140

ISSN 2289-7070

mahasiswa semester kedua dalam kelas kuliah diberi ujian standard geometri iaitu van Hiele Geometric Test (VHGT) yang mengandungi 20 soalan aneka pilihan. Analisis deskriptif data dikotomi dijalankan dengan menggunakan program SPSS dan *Winstep*. Hasil kajian mencadangkan bahawa dalam kalangan guru pra-perkhidmatan, (a) sekurang-kurangnya 7.1% mencapai tahap asas iaitu visualisasi, (b) seramai 5.0% mencapai tahap abstrak, dan (c) selebihnya berada dalam kumpulan yang mempunyai keupayaan tahap analisis kekal serta tidak dapat dibezakan dari aspek kriteria yang digunakan. Kajian ini memberi indikasi bahawa pembinaan dan penilaian semula kandungan buku teks boleh memupuk kesedaran ke arah pemikiran semula tentang kefahaman matematik guru secara kolektif atau secara individu tentang aspek geometri. Situsai ini boleh mencabar struktur kepercayaan lama mereka dan diharapkan mereka mampu menyedari nilai epistemik bentuk dan sifat-sifat bentuk yang berkaitan dengan satu sama lain serta memperoleh pembinaan semula konsep tersebut.

Kata Kunci geometri, tahap geometri van Hiele, keupayaan penaakulan, alat pengajaran, bentuk geometri dua dimensi

BACKGROUND OF THE STUDY

Geometry is in essence linked to the practical study of physical shapes (Atiyah, 1982). Through perception since early childhood, such manifestations are abstracted to form more manipulatable concept of lines and angles alongside the corresponding mental images in favor of visualization or discourses (Gray & Tall, 2007; Fiscbein, 1993). Being adept at visualizing is a requisite for wide range of professions, involving geometric modeling, graphic design, engineering, and teaching, to mention a few. Geometry may therefore help provide an ideal medium for acquiring such visualization skills (Hershkowitz et al., 1990, & NCTM[1], 2000).

Accordingly, a mathematics teacher who is responsible to promote environment for learning must primarily be well-informed not only on the subject matter but also the students' conceptions formed through instructions (Ball et al., 2008). Any idiosyncratic concept images (Vinner, 1991) of which students bear to confront the subject may sometimes impede learning. In order to seek sound arguments used to reshape these conceptions, teachers are urged to develop their reasoning and other higher order thinking skills instead of relying solely on intuitive knowledge (Duval, 1998; Fiscbein, 1993; Schoenfeld, 1992). "Logical reasoning has to be absorbed in the teaching of mathematics so that students can recognize, construct and evaluate predictions and mathematical arguments" (MOE[2], 2006, p. xiii).

While geometry flourishes over the past few decades, the curricular decision on types and depth of geometry content to be taught at school and university levels is yet unresolved (Jones, 2000; Gonzalez & Herbst, 2006). Apart from intuitive and practical geometry, the underlying logical structure is among the numerous aspects of school geometry which has been strikingly trivialized (Hershkowitz et al., 1990). Furthermore, while geometry and reasoning are often downplayed in early education (Clements & Sarama, 2011), the secondary school geometry is taught with more attention focused on algebraic manipulations and algorithmic calculations at the expense of geometrical interpretation of the solution (Mammana & Villani, 1998). The extent to which the role of geometric reasoning may play for such computations is questionable.

More alarmingly, TIMSS[3] 2007 has reported that the lower secondary school (Form Two) Malaysian students did not perform satisfactorily in both domains of content and cognitive (Mullis et al., 2008). Table 1 shows the arranged scale (score) for Malaysian students in Geometry (477) and Reasoning (468) which are found to be below par (500). The data suggested that the item content can hardly be blamed since it covers 11 of 14 of the geometry topics taught in Malaysia. However, the time spent (24%) on teaching the subject is relatively higher than other higher ranking countries such as United States (16%) and Singapore (21%). The overall quality of geometry teaching in Malaysia is questionable.

Table 1 The international ranking in geometry achievement in TIMSS 2007

Ranking	Country	Average scale score		% Topics taught[a]	%Teaching time[b]
		Geometry	Reasoning		
1	Chinese Taipei	592*	591*	76	40
2	Korea, Rep. of	587*	579*	81	34
3	Singapore	578*	579*	71	21
4	Hong Kong	570*	557*	83	31
5	Japan	573*	568*	79	33
7	England	510*	518*	83	21
9	United States	480	505*	78	16
20	Malaysia	477	468	90	24

[a] Average % of students taught the TIMSS math topics.
[b] % of time in math class devoted to geometry during school years.
*above par of 500

Relevantly, teaching practice based solely on the current textbook could be detrimental to learning (van der Sandt, 2007). Geometry content basically encompasses ready-made formulas, imprecise definitions and authoritarian propositions to be memorized by rote (Hershkowitz et al., 1990; Noraini & Tay, 2004). Wu (2005) argued that "the formal reasoning about geometrical configuration has to be conducted entirely on the basis of definition" (p.5). For instance, the adoption of partition classification of various quadrilaterals stresses mainly the meaningless recapitulation of properties without having to define them relationally (Skemp, 1987). The lack of exposure to class inclusion in the long term may hamper the development of logical thinking required for advanced mathematical studies (de Villiers, 1994).

These demands call for substantive pre-service teacher training in the area concerning school geometry. However, any currently offered geometry-based courses are either far too advanced for them to make explicit connections or content irrelevant to teaching school geometry (Hoyles et al., 2001). In the absence of any geometry courses, student teachers are forlornly left to access their own geometry knowledge or recall fragments of school days knowledge. Schoenfeld (1992) was wary of the beliefs about mathematics they experienced during their school days could be transferred into instructional practices.

Significance of Content

Shape and space belong to one of the three main content areas in the Malaysian ICCS[4] (MOE[2], 2006). According to CBMS[5] (2001), pre-service mathematics teachers must "develop competence in basic shapes, their properties, and relationship among them" (p. 21), be able to "make conjectures about geometric shapes and then prove or disprove them" (p. 32), and understand "the nature of axiomatic reasoning and the role that it has played in the development of mathematics" (p. 41) before expecting their students to be able to analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationship (NCTM[1], 2000).

Purpose of the Study

The objective of this study was to examine the extent to which first-year pre-service secondary mathematics teachers have developed their geometric reasoning ability about quadrilaterals and triangles required to teach school geometry. Two following questions are thus addressed through the theoretical lens offered by van Hiele (1999):

1. What are van Hiele levels of reasoning in first-year pre-service secondary mathematics teachers?
2. What are the patterns of difficulty, if any, exhibited with respect to the test content?

Theoretical Framework

The van Hiele model of geometric reasoning was used to frame the study. According to van Hiele (1999) and his proponent researchers (e.g., Battista, 2007; Usiskin, 1982), there were five levels of geometric thinking to be developed sequentially. While only the first four levels were academically relevant to the present study, Level 0 (precognition) was included for those who failed to identify many common shapes (Battista, 2007).

The first four levels are:

Level 1 (visualization) – shapes are syncretically recognized by their visual appearance.

Level 2 (analysis) – shapes are descriptively identified as a bearer of independent properties.

Level 3 (abstraction) – shapes and its properties are logically interrelated through informal deductions.

Level 4 (formal deduction) – shapes and its network of relationships are formally conceptualized in terms of necessary and sufficient conditions.

METHODOLOGY

Participants

A total of 140 second-semester undergraduates (23 male, 117 female) who were enrolled in Bachelor's degree programs in a local university constituted the sample of this study. All undergraduates were majoring in mathematics.

Instrument

The paper-and-pencil van Hiele Geometry Test (VHGT) was constructed as part of the CDASSG[6] project led by Usiskin (1982) in order to test the theory of van Hiele levels. The test which consisted of 20 multiple-choice questions was divided into four subtests (see Table 2) of five successive questions. Based on the theory, each question had been written to operationalize geometric reasoning in terms of behaviors of which learners may exhibit at each level (Usiskin, 1982).

Table 2 Distribution of VHGT questions

	Van Hiele level			
	1	2	3	4
Question number	1 – 5	6 - 10	11 - 15	16 - 20

Two criteria of level assigning were proposed, namely '3 of 5 criterion' and '4 of 5 criterion'. In other words, for the latter, a respondent was said to have mastered a particular level l geometric reasoning if he or she was able to answer at least four of five questions correctly in the corresponding subtest. A respondent was said to perform at a level L of reasoning if he or she had mastered that level and every level $l < L$ while did not master any level $l > L$. Otherwise, his of her responses were considered not fit (Usiskin, 1982). The test was administered to three groups (A, B, C) of undergraduates under naturalistic lecture setting in the early second semester of the academic year 2010/2011. They were given 35 minutes to answer the test questions.

RESULTS

Table 3 shows the crosstabulation analysis of both criteria regarding van Hiele level of geometric reasoning at which the pre-service undergraduates were performing. Current sample consisted of 116 (82.9%) fit and 24 (17.1%) 'not fit' cases. Regardless of which criterion used, there were at least 10 (7.1%) undergraduates performed below the visualization level (Level 1) and at most 7 (5.0%) of them have attained the abstraction level (Level 3). None of them was found performing beyond the latter level. Among these cases, 10 (7.1%) Level 0, 28 (20.0%) Level 1, 42 (30.0%) Level 2, 3 (2.1%) Level 3, and 3 (2.1%) 'not fit' remained consistent across criteria (see diagonal entries). In short, while many undergraduates were ready to see triangles and quadrilaterals as bearers of properties (Level 2), they were still far short of being able to logically order these properties in ways that class inclusion could be followed as an obvious corollary (Level 3).

Table 3 Frequency and percentages of pre-service undergraduates at each van Hiele levels with '4 of 5' criterion (C4) by '3 of 5' criterion (C3)

	Level (C3)					
Level (C4)	0	1	2	3	Not fit	*n*
0	10	12	5	0	5	32
1	–	28	12	1	1	42
2	–	–	42	3	0	45
3	–	–	–	3	–	3
Not fit	–	–	15	0	3	18
N	10	40	74	7	9	140

EDUCATUM - Journal of Science, Mathematics and Technology Vol. 2 No.1 (2015) 1 - 12
ISSN 2289-7070

The findings can be made more interpretable when any off-diagonal cases (n = 54, 38.6%) are viewed as the level reduction effects after employing the stricter '4 of 5' criterion compared to '3 of 5' criterion. Such decline was expected insofar since it seemed harder to pass the subtests with the higher cutscore. The most striking deviation occurred at Level 2. Among 74 (52.9%) Level 2 candidates, 12 (16.2%) and 15 (20.3%) had slipped to Level 1 and 'not fit' category respectively. For fit cases, modal response analysis (see Table 4) revealed that the first subtest scores ranged from four to five while all falling one short to meet the stricter criterion in second subtest. However, this situation was reversed for the 'not fit' cases. Likewise, three Level 3 candidates who had correctly answered three questions in the third subtest slipped to Level 2. Hence, it was interesting to identify any questions of which undergraduates most likely failed to answer and hence causing them fell short of higher level when '4 of 5' criterion was used.

Table 4 Modal response and frequency of off-diagonal Level 2 cases with '3 of 5' by '4 of 5' criteria

	Level (C4)					
Level (C3)	0	*n*	1	*n*	Not fit	*n*
2	(3-3-0-0)	1	(4-3-0-0)	2	(3-4-0-1)	2
	(3-3-1-0)	2	(4-3-1-0)	2	(3-4-0-2)	1
	(3-3-1-1)	1	(4-3-1-1)	3	(3-4-1-0)	3
	(3-3-2-0)	1	(4-3-1-2)	1	(3-4-1-1)	1
			(4-3-2-0)	1	(3-4-1-2)	1
			(5-3-1-0)	1	(3-4-2-0)	2
			(5-3-1-1)	1	(3-5-0-0)	2
			(5-3-2-1)	1	(3-5-0-1)	1
					(3-5-1-0)	1
					(3-5-2-0)	1

Note. The modal response is the parenthesized four-number chain (a-b-c-d) where a, b, c, d are the number of correct response in subtests 1, 2, 3, 4 respectively. C3 = '3 of 5' criterion; C4 = '4 of 5' criterion.

The Rasch-based *Bond & Fox Steps* version of *Winsteps* software was used to diagnose any problematic items as well as their location on a linear interval scale called *logit* (Bond & Fox, 2007). This is an unbounded scale on which both the undergraduate performances and question difficulties can be plotted concomitantly (see Figure 1). As one goes up the scale, questions become more difficult and individuals became more able. The right panel of Figure 1 showed that the cluster of questions represented a hierarchical response structure except the anomalous questions 5 (Level 1), 13 and 14 (both Level 3) with item measures 0.36, 2.06 and 3.12 respectively, were found far higher than the average difficulty within each level (see Table 5).

Table 5 Difficulty threshold for VHGT questions

	Level 1					Level 2				
Questions	1	2	3	4	5	6	7	8	9	10
Difficulty threshold	-2.87	-1.08	-3.81	-2.15	**0.36**	-1.68	-0.67	-0.38	-1.64	-0.71

Table 5 continued

M			-1.91					-1.02		
			Level 3					Level 4		
Questions	11	12	13	14	15	16	17	18	19	20
Difficulty threshold	1.10	**-0.45**	**2.06**	**3.12**	0.22	1.48	2.06	1.68	1.98	1.37
M			1.21					1.71		

For illustration, question 13 (Level 3) was seen as hard as question 17 (Level 4) and harder than any questions below the difficulty threshold 2.06 but relatively easier compared to (the most difficult) question 14. This also validated the earlier conviction that most undergraduates failed to reach Level 3 despite their success on the rest of questions (11, 12, 15). Likewise, analysis of response pattern exhibited by 12 Level 2 candidates to three harder questions (7, 8, 10) showed that 10 (83.3%) of them had answered at least one of these question wrongly.

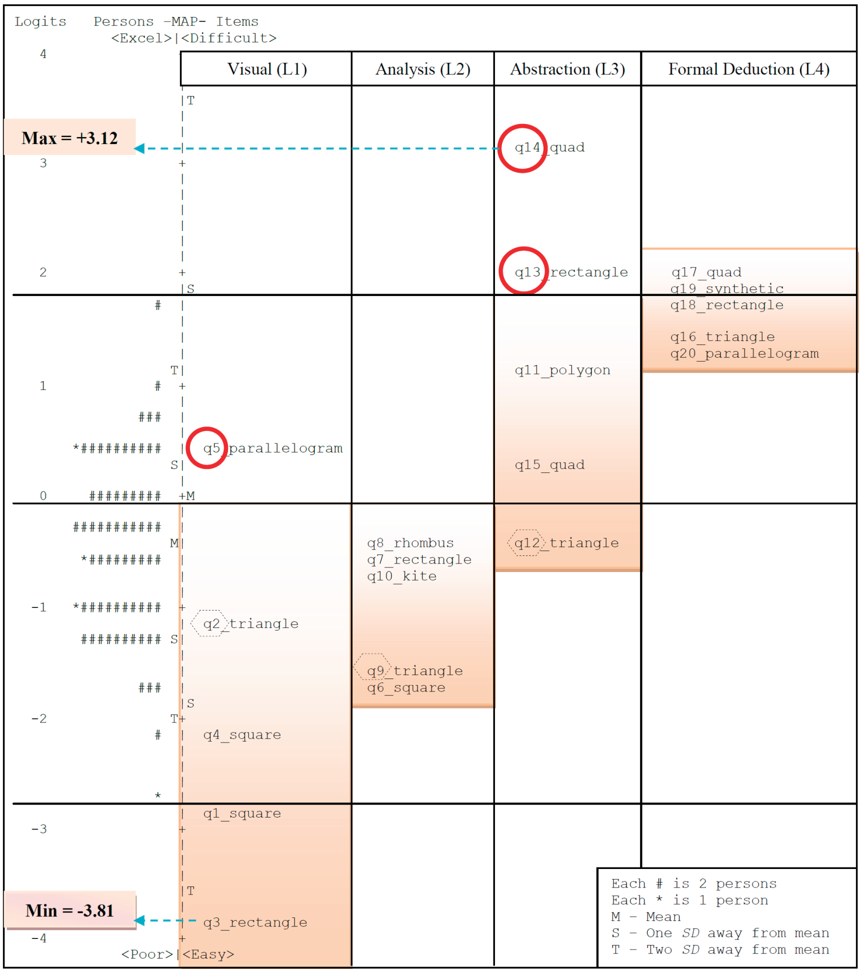

Figure 1 Person-question distribution map: Difficulty threshold of questions in each van Hiele level

More interestingly, the common content area underlying three out-of-level questions (5, 13, 14) seemed to correspond closely to the class inclusion of quadrilaterals (parallelograms, rectangle, rhombus and squares). Questions (17 and 18) regarding concept of sufficient and necessary condition of quadrilaterals also equally troubled them. On the other hand, except for question 16, they dealt more comfortably and with questions (2, 9, 12) that focused solely on triangle concept as evident from Figure 1 and that question 12 (of Level 3) receded below average difficulty.

DISCUSSION AND CONCLUSION

The result shows (see Table 3) a relatively higher number of Level 1 and Level 2 responses that signifies most pre-service undergraduates' geometric reasoning ability have attained the analysis level (Level 2) – a result consistent with those of Lawrie's (1999) study. In other words, they are considered competence in discriminating various triangles and quadrilaterals as well as identifying the critical attributes which characterize each class of shapes.

Despite being in early stage of training program, it was dubious if such capacity may develop over time into illuminating a network of relationship linking the properties and deducing a property on the basis of another (Level 3), especially in the absence of any school geometry-based courses. Van Hiele (1999) argued that progress through higher level of reasoning did not depend heavily on factors other than specific instructional experience – a claim which had been verified by many scholars (e.g., Aydin & Halat, 2009). This has called for the need to rethink the mathematics curriculum offered to the current cohort of pre-service undergraduates.

In addition, further analysis of 'not fit' category revealed that 18 (75.0%) of the 24 cases concerning the failure of shapes recognition (Level 1) while showing competence in identifying the properties of these shapes (Level 2) – an anomaly according to van Hiele theory. Besides being troubled with question 5 (Level 1) whose difficulty threshold was

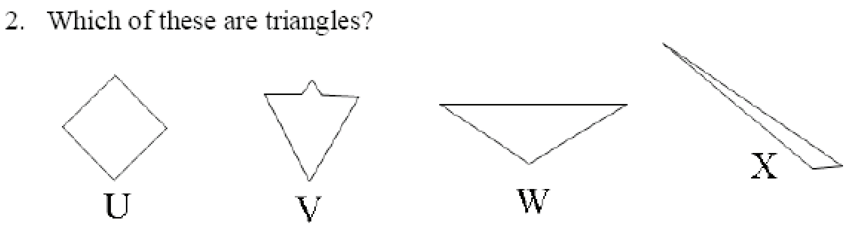

Figure 2 VHGT question 2: Visual images of triangles

almost similar to question 15 (Level 3) the response of question 2 took us by surprise: 47 (33.6%) denied shape X and 3 (2.1%) undergraduates disregarded all shapes available as a triangle (see Figure 2). Prototypical images of triangles might be the most accountable reason to such disclaims (Hershkowitz et al., 1990). The implication was to inform the importance of knowledge building on the undergraduates' partial visual images before making them accessible to their future students. "The earlier students learn proper usage

ISSN 2289-7070

of vocabulary and syntax, the further they would travel in van Hiele hierarchy of learning" (Noraini, 2006, p. 83).

Accordingly, the language of geometry used is of prime importance in reasoning: "Every level has its own linguistic symbol and its own network of relationships connecting these symbols" (Usiskin, 1982, p. 5). Any incompatible 'symbol' such as the quantifier 'for all' used in test may result in confusion, as evident from question 14 (see Figures 3). It also suggested that questions focusing on the generality were much harder for them to tackle (Lawrie, 1999).

14. Which is true?

A. All properties of rectangles are properties of all squares.

B. All properties of squares are properties of all rectangles.

C. All properties of rectangles are properties of all parallelograms.

D. All properties of squares are properties of all parallelograms.

E. None of (A) - (D) is true.

Figure 3 VHGT question 14: How do classes of squares, rectangles and parallelograms associate to each other?

Although rectangles and squares can look somehow different in appearance (Levels 1 and 2), such a partition classification may overshadow the existence of interrelationships between the shapes (Level 3). Questions 13, 14 and probably question 5 were among exemplars to explain such hurdle. Among 127 (90.7%) undergraduates who failed on question 13, 117 (83.6%) did not think the square P can be a rectangle whereas 7 (5.0%) only considered the prototypical 'vertical' rectangle Q (see Figure 4). This way of thinking

13. Which of these can be called rectangles?

A. All can.

B. Q only

C. R only

D. P and Q only

E. Q and R only

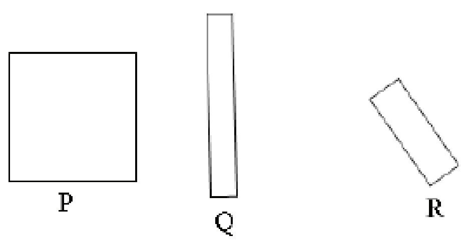

Figure 4 VHGT question 13: Can a square be rectangle?

was almost isomorphic to the case of question 5: Among 94 (67.1%) of them who failed, 70 (50.0%) omitted the 'diamond-like' rhombus L as parallelogram and 21 (15.0%) only accepted the 'standard-oriented' parallelogram J whose two long sides were laid horizontally (see Figure 5).

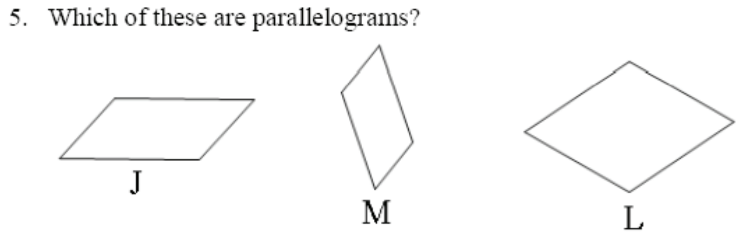

Figure 5 VHGT question 5: Visual images of parallelogram

The non-salient attribute (two long sides and two short sides) of prototypical rectangle and parallelogram was thought to be responsible for evoking the dominant visual differences that hindered class inclusion. Despite being prone to overlook, ‘can be’ and ‘are’ in questions 13 and 5 might be interpreted semantically similar by undergraduates such as “since there is no rectangle whose adjacent sides *can be* made equal, squares *are* thus not rectangles.” Nevertheless, the class inclusion could be a matter of intention rooted in beliefs. The lack of such intention barely rendered them any meaningful definitions supportive of class inclusion (Shir & Zaslavsky, 2002).

The result can also leave much room for speculation about textbook-driven instructional practice. Lee (2006) suggested that reconstruction and reevaluation of textbook content may foster awareness towards rethinking about the teachers own mathematical dispositions. By collectively or individually challenging their old belief structure, it is hoped that they could realize the epistemic value of having the shapes and its properties be related to each other as well as gaining ownership of such reconstructions (de Villiers, 1994). Nevertheless, a joint effort from several parties is needed to empower the pre-service teachers to be more independent, flexible and strong-minded in order to meet the goals addressed by NCTM (2000) and CBSM[5] (2001).

Acronyms

[1] NCTM – National Council of Teachers of Mathematics
[2] MOE – Ministry of Education
[3] TIMSS – Trends in International Mathematics and Science Study
[4] ICCS – Integrated Curriculum for Secondary Schools (Mathematics)
[5] CBMS – Conference Board of the Mathematical Sciences
[6] CDASSG – Cognitive Development and Achievement in Secondary School

REFERENCES

Atiyah, M. (1982). What is geometry. *The Mathematical Gazette, 66*(437), 179-184.

Aydin, N., & Halat, E. (2009). The impacts of undergraduate mathematics courses on college students’ geometric reasoning stages. *The Montana Mathematics Enthusiast*, *6*(1-2), 151-164.

Ball, D. L., Thames, M. H. & Phelps, G. (2008). Content knowledge for teaching: What make it special? *Journal of Teacher Education*, *59*(5), 389-407.

Battista, M. T. (2007). The development of Geometric and Spatial Thinking. In F. K. Lester (Ed.), *Second Handbook of Research on Mathematics Teaching and Learning* (pp. 843-908). Charlotte, NC: Information Age Publishing.

Bond, T. G., & Fox, C. M. (2007). *Applying the Rasch model: Fundamental measurement in the human sciences* (2nd ed.). Mahwah, N.J.: Erlbaum.

Clements, D. H., & Sarama, J. (2011). Early childhood teacher education: The case of geometry. *Journal of Mathematics Teacher Education*, *14*, 133-148. doi:10.1007/s10857-011-9173-0

Conference Board of the Mathematical Sciences. (2001). *The mathematical education of teachers*. Providence, RI and Washington DC: American Mathematical Society and Mathematical Association of America.

de Villiers, M.D. (1994). The role and function of a hierarchical classification of quadrilaterals. *For the Learning of Mathematics, 14*(1), 11-18.

Duval, R. (1998). Geometry from a cognitive point of view. In C. Mammana & V. Villani (Eds.), *Perspectives on the teaching of Geometry for the 21st Century* (pp. 37-52). Dordrecht/Boston: Kluwer Academic Publishers.

Fischbein, E. (1993). The theory of figural concepts. *Educational Studies in Mathematics, 24*, 139–162.

Gonzalez, G. & Herbst, P. G. (2006). Competing arguments for the geometry course: Why were American high school students supposed to study geometry in twentieth century? *International Journal for the History of Mathematics Education*, *1*(1), 7-33.

Gray, E. M., & Tall, D. O. (2007). Abstraction as a natural process of mental compression. *Mathematics Education Research Journal*, *19*(2), 23-40.

Hershkowitz, R., Ben-Chaim, D., Hoyles, C., Lappan, G., Mitchelmore, M. C., & Vinner, S. (1990). Psychological aspects of learning geometry. In P. Nesher & J. Kilpatrick (Eds.), *Mathematics and cognition: A research synthesis by the International Group for Psychology of Mathematics Education* (pp. 70-95). Cambridge, MA: Cambridge University Press.

Hoyles, C., Newman, K., & Noss, R. (2001). Changing patterns of transition from school to university mathematics. *International Journal of Mathematical Education in Science and Technology*, *32*(6), 829-845.

Jones, K. (2000). Issues in the teaching and learning of Geometry. In L. Haggarty (Ed.), *Aspects of Teaching Secondary Mathematics: Perspectives on practice* (pp. 121-139). London: RoutledgeFalmer.

Lawrie, C. (1999). Exploring van Hiele levels of understanding using a Rasch Analysis. In O. Zaslavsky (Ed.), *Proceedings of the 23rd PME International Conference*, *3*, 209-216.

Lee, K. H. (2006). Evaluation and reconstruction of mathematics textbook by prospective teachers. In J. Novotna, H. Moraova, M. Kratka & N. Stehlikova (Eds.), *Proceedings of 30th Conference of the International Group for the Psychology of Mathematics Education*, Vol. 4 (pp. 41-48). Prague, Czech Republic.

Mammana, C., & Villani, V. (1998). Perspective on the teaching of geometry for the 21st century. *Educational Studies in Mathematics*, *28*, 91-98.

Ministry of Education. (2006). *Integrated curriculum for secondary schools: Curriculum specifications; Mathematics Forms 1, 2, 3, 4 & 5*. Kuala Lumpur: Curriculum Development Centre.

Mullis, I.V.S., Martin, M.O., & Foy, P. (2008). TIMSS 2007 International Mathematics Report: Findings from IEA's Trends in International Mathematics and Science Study at the Fourth and Eighth Grades. Chestnut Hill, MA: TIMSS & PIRLS International Study Center, Boston College.

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA: NCTM.

Noraini Idris. (2006). *Teaching and learning of mathematics: Making sense and developing cognitive abilities*. Kuala Lumpur: Utusan Publications & Distributors.

Noraini Idris & Tay, B. L. (2004). Teaching and learning of geometry: Problems and prospects. Majalah Pendidikan, *27*, 165-178.

ISSN 2289-7070

Schoenfeld, A. H. (1992). Learning to think mathematically: Problem solving, metacognition, and sense-making in mathematics. In D. Grouws (Ed.), *Handbook for Research on Mathematics Teaching and Learning* (pp. 334-370). NY: MacMillian.

Shir, K. & Zaslavsky, O. (2002). Students' conceptions of an acceptable geometric definition. In A. D. Cockburn & E. Nardi (Eds.), *Proceedings of the 26th Conference of the International Group for the Psychology of Mathematics Education, Vol. 4*, 201-208.

Skemp, R. R. (1987). *The psychological of learning mathematics*. Hillsdale, NJ: Lawrence Erlbaum.

Usiskin, Z. (1982). *Van Hiele levels and achievement in secondary school geometry*. (Final report of the Cognitive Development and Achievement in Secondary School Geometry Project). Chicago: University of Chicago. (ERIC Document Reproduction Service No. ED 220 288).

van der Sandt, S. (2007). Pre-service geometry education in South Africa: A typical case. *IUMPTS: The Journal* (*Content Knowledge*), *1*, 1-9. Retrieved from http://www.k-12prep.math.ttu.edu/

Van Hiele, P. M. (1999). Developing geometric thinking through activities that begin with play. *Teaching Children Mathematics*, *5*(6), 310-316.

Vinner, S. (1991). The role of definitions in the teaching and learning mathematics. In D. Tall (Ed.), *Advanced Mathematical Thinking* (pp. 65-81). Kluwer, The Netherlands: Dordrecht.

Wu, H. (2005, September). Key *mathematical ideas in Grades 5-8*. Paper presented at the NCTM Annual Meeting, Anaheim, CA.