International Journal of Assessment and Evaluation in Education
Vol. 3/Dec 2013 (31-45) ISSN 2232-1926

The Functions of Graphics and Visual Reasoning Demand in Mathematical Problems in the Malaysian National Examination

Haliza Abd Hamid
Universiti Malaya, Malaysia
Noraini Idris
Universiti Malaya, Malaysia
and Universiti Pendidikan Sultan Idris, Malaysia (Secondment)
Ruzela Tapsir
INTEC Education College, Shah Alam, Selangor, Malaysia

Abstract

Evaluation on the quality of education in Malaysia is assessed mainly through written examinations. Both the Malaysian Education Development Plan for 2001-2010 and 2013-2025 outlined restructuring of the Integrated Secondary School Curriculum (KBSM) with one of the aims to enhance students' critical and creative thinking which could be achieved through the imperative visual reasoning skills in mathematics education specifically in the solving of mathematical problems. This study presents the analysis on the roles of graphical tasks, which are visual in nature, in the mathematics papers, Mathematics and Additional Mathematics, of the main Malaysian national examination, the Sijil Pelajaran Malaysia, with the visual reasoning demand as the point of focus. A quantitative analysis is carried out on the role of graphics: decorative, representational, organizational or informational, and the trend in topics with graphic-accompanied tasks. In addition, the level of visual reasoning demand by the graphical tasks: reading the graphics, reading between the graphics and reading beyond the graphics are also looked into. Findings observe that various forms of graphics are over-represented in the examination papers and suggest that the students' ability to read and interpret graphics is central to the choice and structure of graphics accompanied in the problems. How information is presented plays a critical role in establishing students' understanding of the mathematical concepts.

Keywords *mathematic education, visual reasoning, graphics, mathematical tasks, decoding*

INTRODUCTION

"Graphic" is the term used by Cox and Grawemeyer (2003) to describe a visual representation that can be utilized to reason and solve mathematical problems. Graphics include any non-verbal or non-sentential forms such as diagrams, charts, graphs, maps, tables etc. They are visual in nature and due to their efficiency in solving problems as compared to words and verbal capacity (Larkin & Simon, 1987; Hembree, 1992), they are employed in various subjects and courses at schools and various disciplines at higher educational levels (Blackwell & Engelhardt, 2002). The use of graphics as mathematical thinking tools is recognized by educators and researchers due to their use to simplify complicated situations, and to confirm more abstract concepts (Diezmann & English, 2001; Novick & Hurley, 2001; Carney & Levin, 2002; Kidman, 2002; Pantziara, Gagatsis & Pitta-Pantazi, 2004; Cook, 2011).

The emergence of graphics in school mathematics

Students need to deal with a lot of reasoning and logic in mathematical problems and therefore their solving ability is important in mathematics education. However, the development of such special ability is difficult for students. Therefore, various teaching and learning methods are proposed to overcome the problems, one of which is the use of graphics and consequently the ability to reason visually. Due to empirical results of successful use of graphics in various aspects of problem solving (Larkin & Simon, 1987; Mayer, 2003; Ainsworth & Loizou, 2003; Program for International Student Assessment (PISA), 2004), mathematics teachers and educators have mostly employed graphics in their classroom teaching environments to support students understanding the concepts. Graphics–accompanied tasks should provide experiences to students such that they are able develop understanding based on visual reasoning and to confront their misinterpreting of graphics (Shaughnessy, 2003).

The last decade has witnessed graphics to emerge as important elements for all students to master, particularly those in related fields such as architecture, engineering and design. From the mathematical prospect, the main objective of using graphics is the understanding of the core abstract and formal concepts. Unfortunately, many research identified that students are lacking in the use of graphics in their worked solutions. This could be due to their inability or nonconfidence to construct appropriate or correct graphics for the problem to represent the situation. Graphics are usually complex and loaded with information, and therefore are more effective since they explicitly show important conceptual links among parts of information (Cook, 2011; Blackwell & Engelhardt, 2002; Kidman, 2002; Ainsworth & Loizou, 2003). Diezmann (2000) and Uesaka and Manalo (2011) described the complicated cognitive process involved and the convention used for the graphics prohibit students from opting them to aid for solutions (Novick, 2006) which in turn require students to have the ability to understand and interpret them effectively. The lack of this skill together with their perception about the efficacy and difficulties of their usage may contribute to the reluctance to use graphics (Uesaka & Manalo, 2011).

Taking graphics as unreal (Larkin & Simons, 1987), students fail to read or extract relevant data or information, which consequently causes them to revert to textual explanation for clarifications, assuming text alone provide all the required information (Winn, 1987; Paoletti, 2004; Koay, 2007). Many (Dufour-Janvier et al, 1987; Ichikawa, 2000; Novick, 2006; Uesaka, Manalo & Ichikawa 2007, 2010) blamed this as leading to students oblivious of graphics efficiency. The mode of how to display the information is critical in students' sense-making and interpreting them, for this may cause potential conflict between conceptual and perceptual features of the graphics (Bardelle, 2010). Working with graphics requires a continuous interplay between the graphics system (Lowrie & Diezmann, 2007) and the accompanying texts involved in the statement, usually verbal texts or symbolic expressions. A substantial body of literature has indicated difficulties students faced even with the most typical problems involving graphics and embedded information. They either utilized inefficient or incompetent techniques and strategies or were unable to see the relationships between concepts among parts of the diagrams (Lowrie & Diezmann, 2007). Postigo and Pozo (2004, p. 628) had also proposed that, "students restrict themselves to reading data and processing specific aspects of the material and encounter problems when they have to go beyond this elementary level and interpret the information represented".

THEORETICAL FRAMEWORK

The design of graphics is essential in assisting students' engagement in mathematical problems and encouraging visual reasoning (Francisco & Maher, 2005). Henningsen and Stein (1997) discovered that less complex visual tasks reduce students' cognitive ability while those with more complicated graphical tasks are able to challenge students to reason effectively. Carney and Levin (2002) proposed four functions that graphics aid in the accompanying texts of mathematical problems: decorative, representational, organizational and informational. Decorative graphics, as the term imply, just serve as 'escort' to the textual questions. They do not supply any information to help in the solving process or in other words, they contribute very insignificant effect. Students may opt to use the analytical method or they may strategize using the visualization technique, for example, by drawing their very own diagrams, pictures or sketches to assist them in solving the problem. Representational graphics depict part(s) of the information in the problem for reference. Students should be able to utilize their visualization skills to help them cross-check the solution for the problems. Organizational graphics guide students in sequence of steps to undertake to reach solutions. Although the graphics provided should be in the form of 'direct help' for the students, they may restructure or make adjustments to the graphics either in drawing form or in their mind, to suit the solutions process. In informational graphics, the whole problem is 'in' the graphics, without which students are unable to proceed solving it. The students need to manage their visual thinking actively to search and relate information depicted in the graphics. They may also apply the 'trial and error' method by manipulating part(s) or the whole graphics. In the case where students opted to work with algebraic expressions, the informational graphics provided could support any disagreement or inconsistencies of the solutions.

The importance of graphics in the teaching and learning of mathematics is widely recognized. However, although educators and mathematicians utilize a vast range of graphics for mathematical communication and analysis (Harris, 1996), there has been limited attention to the interrelationship between numeracy and graphics (Pugalee, 1999). This relationship involves the ability to encode mathematical information into graphics and to decode mathematical information from graphics (Baker, Corbett, & Koedinger, 2001). Students develop 'graphics sense' gradually as a result of creating graphics and using already designed graphics in a variety of problem contexts that require making sense of the problems (Friel, Curcuio & Bright, 2001). Similarly, Murray, Kirsh and Jenkins (1997) define document literacy to contain processes of locating, integrating and generating tasks as the knowledge and skills required to trace and to use information contained in various formats including graphics. In Friel, Curcio and Bright (2001), 'reading the data' phase of investigating graphics focuses on extracting data from graphics and requires the locating and translating of information based on specific rules or conditions (Murray et al., 1997) as a form of communication. The phase of 'reading between the data' focuses on interpolating and finding connection among the data shown in the graphics. Students are to integrate or pull together two or more pieces of information (Murray et al., 1997) and to observe relationships among them in order to interpret the problems. The last and advance phase of 'reading beyond the data' focuses on extrapolating information and analyzing the relationships implicitly out of the data shown in the graphics for the students to generate, predict and make inferences. To extrapolate, students need to extend the interpreting phase by stating not only the essence of the communication but to identify some of the consequences through noting the trend perceived in the data or specifying implications that are also are based on personal background knowledge (Murray et al., 1997).

Texts and graphs are of different cognitive representations and required the brain to process them through separate memory systems (Paivio, 1991), the verbal memory and image memory respectively. When either of them is attained from the sensory, they are shoved separately to their respective processor, the verbal processor or the visual processor. The important and crucial point is when their path crossed such that one is able to stimulate and inspire the other. For example, consider the word 'quadratic' or the mathematical function $y = ax^2 + bx + c$. Upon encountering the word, those who had learned quadratics theory will promptly 'see' a parabolic shape or curve. Both the word and curve come together to the mind because they are related and meaningful.

OUR STUDY

The present article reports the first phase of a larger work, to provide the baseline data on visual reasoning using graphs at the pre-university level. Core subject such as mathematics (and science) must strictly adhere to the learning outcomes designed by the curriculum developer in order for students to be competent of their knowledge gained. The previous Malaysian Education Development Plan 2001-2010 aimed : *To improve the quality of secondary education..to review and strengthen the secondary school curriculum...Emphasis will be given to the development of academic skills, especially in mastering skills of learning, communication, critical and creative thinking... The secondary education curriculum will strengthen intervention programmes for students with learning problems, integrate ICT in the curriculum*...(p.6). Recently, the newly launched Malaysian Education Development Plan 2013-2025 strategizes an upward shift of the teaching and learning of mathematics (together with engineering, technology, science and language) to an international-level quality. Inclusive in the plan is to review the standardized curriculum of the primary and secondary schools. Students will be required to be equipped with various cognitive skills such as reasoning, creative, critical and innovative thinking in order to prepare them to deal with real life situations.

Research had evidence that visual thinking and visualization ability are among the best methods for students to grasp concepts. Therefore, we have proposed to consider the graphics in the mathematics papers of the main Malaysian national examination, Sijil Pelajaran Malaysia as the main focus of our present study. Graphics are often regarded as trivial in either textbooks or examination papers and consequently they are of limited use (Dhakulkar & Nagarjuna, 2011). Since graphics have a marked influence on how concepts are understood, it is important to investigate the role they play and the forms they appear that many mathematics teachers implement in the classroom and consequently in the examination contexts. Specifically, the purpose of the study was to investigate the function played by graphics utilized in the examinations and their reasoning level demanded in order to solve the accompanied tasks. Accordingly, the study addressed the following research questions: First, what are the roles of the graphics in accompanying texts to solve mathematical tasks? Second, how is the trend of occurrence of graphics among topics in the mathematical examination papers? Third, what is the visual reasoning level required of the graphics to solve mathematical tasks?

METHODOLOGY

In the Malaysian Education System, examinations are the main assessment criteria that determine the educational success of students. The Sijil Pelajaran Malaysia, taken by students at the end of their secondary school is in fact the basis of awarding scholarships to further their studies to higher levels of education. Therefore, it is imperative that examination questions and tasks are well-designed to extract students' understanding of concepts and reasoning as outlined in the learning objectives of the Curriculum Development Centre (2003). For this reason, we have opted for both the Mathematics (Paper 1 and 2) and Additional Mathematics (Paper 1 and 2) papers. For the purpose of the analysis, we combined Paper 1 and Paper 2 of each subject. We covered all examination papers for the year 2005 until 2012 and thus have a total of 32 sets of papers in our sample. All the mathematical formulae, instructions for candidates and questions were written in English followed by the Malay version, to help clarify information for those who are not very well-versed with the English language. The summary of the format and contents of both mathematical papers is shown in Table 1.

Table 1 The summary of the format and content of the Mathematics and Additional Mathematics papers

Subject		**Time (hour)**	**No. of questions**	**Nature of questions**	**Total marks**
Mathematics	Paper 1	1 ¼	40	Multiple choice (4 options)	-
	Paper 2	2 ½	Section A : 11	Subjective & straight to the point on basic and tangible concepts	52
			Section B : 5 (to answer 4)	Subjective and more structured of complex and abstract concepts	48

Table 1 *(cont.)*

Additional Mathematics	Paper 1	2	25	Subjective and straight to the point on basic and tangible concepts	80
	Paper 2	2 ½	Section A : 6	Medium-length, subjective and structured on basic and tangible concepts	40
			Section B : 4 (to answer 4)	Long, subjective and structured on basic and tangible concepts	40
			Section C : 4 (to answer 2)	Long, subjective and structured on applications of concepts	20

We use the term *graphics* to refer to any non-written elements that have been displayed to accompany the tasks with the intention to be part of the instructions or information to solve the problems. In categorizing the functions of graphics embedded in the tasks, we first tried out the verbal or written instructions without referring to the graphics. The graphics were classed into their respective category depending on whether or not or how much help was needed to assist the verbal information. Samples of each category were validated by experts in the area to confirm our decision. To answer the second research question, we needed to group the topics taught into common strands. Based on our discretion and experience in teaching the topics and dealing with setting questions for assessments such as assignments, quizzes and tests, we listed the topics taught in the subject Mathematics and Additional Mathematics as in Table 2 and Table 3 respectively. We then identified the number and calculated the percentages of graphics employed for each strand. For the third part of the analysis, we presented samples of questions for each of the higher levels of visual reasoning adopted by the problems. Again, prior to the selection of the levels, samples of the work were sent to experts in the areas for validation.

Table 2 The strands of topics for the SPM Mathematics paper

Algebra	**Functions & Graphs**	**2-Dimensional Problem**	**3-Dimensional Problem**	**Statistics**
Number base Algebraic Expression Algebraic formulae Indices Linear inequalities Standard form Matrices Variations Mathematical reasoning	Linear equation The straight line Quadratic expression & equations Graphs of functions Gradient & area under graph	Polygon Transformation Trigonometry Circles Angles of elevation & depression Bearing	Line & Planes Earth as a sphere Solid geometry Plans & elevations	Sets Probability Statistics

Table 3 The strands of topics for the SPM Additional Mathematics paper

Algebra	Functions & Graphs	Coordinate Geometry & Vectors	Trigonometry	Calculus	Statistics
Indices & Logarithms Index numbers Progressions Linear law	Functions Quadratic equations & functions Simultaneous equations Linear programming	Coordinate geometry Vectors	Circular measure Solution of triangles Trigonometric functions	Differentiation Integration Motion along a straight line	Permutations & combinations Probability Probability distributions Statistics

RESULTS AND TRENDS

We wanted to see how graphics were used in students' understanding of mathematical concepts, therefore we wished to get a pattern on the role of graphics used in the examination situations. Table 4 shows the percentages of questions with graphics in both the Mathematics and Additional Mathematics papers for the years 2005 until 2012. Note that the Mathematics papers have more than 50% of questions that contain graphics yearly. Although the Additional Mathematics papers have quite lower percentages, the compositions are still distinguishable since at least 30% of the questions are graphic-dependent.

Table 4 Percentages of graphics appearance in the SPM Mathematics and Additional Mathematics papers

Year	Mathematics	Additional Mathematics
2005	59	48
2006	57	50
2007	52	35
2008	61	38
2009	57	35
2010	52	43
2011	59	45
2012	57	40

Table 5 displays the distribution of the graphics used in both the Mathematics and Additional Mathematics papers with their respective functions: decorative (D), representational (R), organizational (O) and informational (I).The percentages were calculated out of those questions with graphics only. It can be seen that the graphics function of communicating information plays the biggest role with at least 50% every year and in both papers. The decorative functional graphics contributed in small percentages with less than 20% each for both papers. They can be considered as 'ineffective' resulting in their very minimal use in the papers. On the other hand, those graphics with representational and organizational functions make significant appearances between 21% and 56% in the Additional Mathematics papers as compared to smaller percentages of 12% to 22% in the Mathematics papers. The Additional Mathematics papers are making used of more representational and organizational graphics due to the longer and application nature of the problems posed. In both papers, the informational graphics play bigger roles in accompanying the verbal information. Thus the trend indicates that graphics are over-used in the examinations which indirectly reflect the amount that should be employed in the teaching and learning or classroom context.

Table 5 Percentages of function of graphics in the SPM Mathematics and Additional Mathematics papers

Year	Mathematics				Additional Mathematics			
	D	**R**	**O**	**I**	**D**	**R**	**O**	**I**
2005	6	18	18	67	16	47	21	50
2006	3	16	16	78	0	3	25	55
2007	3	21	17	72	7	50	29	57
2008	9	18	12	71	13	40	33	67
2009	3	22	13	66	14	36	29	71
2010	0	21	21	79	6	47	29	65
2011	6	15	18	67	0	50	28	67
2012	6	19	13	69	6	56	38	63

Note: D = Decorative, R = Representational, O = Organizational, I = Informational

The samples of questions with different functions of the graphics are as shown in Figure 1. For the purpose of the study, only the English versions of the questions are shown here. The sample of a decorative graphic is as shown in Figure 1(a), the graphic showing the cylinder inscribed in the cone serves no more than just a visualization aid or it may help 'to ease tension' in reading the question. The diagram in Figure 1(b) is able to help students to represent the actual situation of the problem. Those students who are not familiar with some mathematical terminologies may make reference to the representational graphics for assistance in solving the problems. In this particular task, students' visualization and spatial reasoning can be enhanced through the respective location and height of the pole and the related angle of elevation. The function of the organizational graphic as shown in Figure 1(c) helps students to strategize the sequence of steps based on the information tabulated i.e. the complete data for stationery Q and the data provided for the other stationeries provide patterns and guide the students to work on the values of x, y and z. It is also observed that most of the organizational graphics in our sample are in table or chart forms. The informational graphics, as shown in Figure 1(d) is a must to the problem since the data needed for the solution process are contained in the accompanying graphics. Informational graphics are able to help students organize their strategy and thinking skills and subsequently assist them to act as feedback or to double-check the solutions obtained. Alternatively, students may opt to use the trial-and-error method. As in the example, students must realize the position of the line PTS with respect to the regular hexagon PQTUVW and the size of angle R in order to solve for the value x.

Diagram 3 shows a solid cone with radius 9cm and height 14 cm. a cylinder with radius 3 cm and height 7 cm is taken out of the solid.

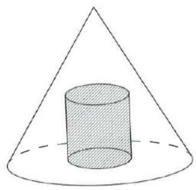


Diagram 3

Calculate the volume, in cm^3 of the remaining solid.

(a) Decorative: Mathematics 2010, Q29

Diagram 8 shows a vertical pole RQ. P is a point which is 20 m from Q on a horizontal plane. The point R is vertically above the point Q.

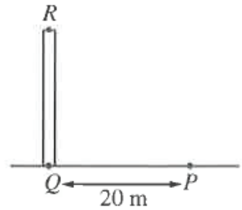


Diagram 8

The angle of elevation of point R from point P is 44^o.
Calculate the height, in m, of the pole.

(b) Representational: Mathematics 2010, Q15

Table 13 shows the prices, the price indices and weightages for four types of stationery P, Q, R and S.

Stationery *Alat tulis*	Price (RM) per unit *Harga (RM) per unit*		Price index in the year 2008 based on the year 2007 *Indeks harga pada tahun 2008 berasaskan tahun 2007*	Weightage *Pemberat*
	Year 2007 *Tahun 2007*	Year 2008 *Tahun 2008*		
P	2.80	2.10	*x*	4
Q	4.00	4.80	120	2
R	2.00	*y*	130	3
S	*z*	5.80	116	*m*

a. Find the value of x, y and z.
b. The composite index for the price of the stationary in the year 2008 based on the year 2007 is 108.4. Calculate the value of m.
c. The total expenditure for the stationery in the year 2007 is RM525.Calculate the corresponding total expenditure in the year 2008.
d. The price index for Q in the year 2009 based on the year 2007 is 132. Calculate the price index for Q in the year 2009 based on the year 2008.

(c) Organizational: Additional Mathematics 2009, Q13

In Diagram 1, PQTUVW is a regular hexagon. PTS and PQR are straight lines.

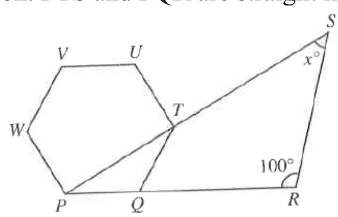


Diagram 1

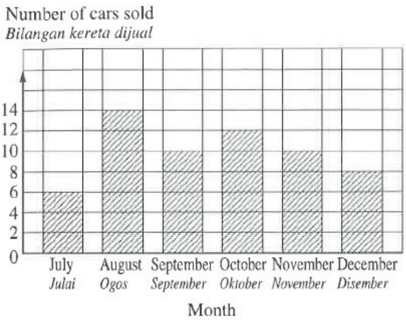
Find the value of x.

(d) Informational: Mathematics 2010, Q7

Figure 1 Samples of questions with different functions of graphics used in the mathematical examination papers

In our study, we identified that some of the graphics are of dual-functions such that they are well-organized to direct the solution process and at the same time students need to refer to the graphics for required information the in Figure 2(a). Another example as in Figure 2(b) shows that the graphic serves both the informational and representational functions.

Diagram 13 is a bar chart showing the number of cars sold by a salesman from July to December 2009.



The number of cars sold from July to December is 20% more than the number of cars sold from January to June. The commission earned by the salesman for each car sold was RM1500. Calculate the total commission paid by the company to the salesman in that year.

Organizational & Informational : Mathematics 2010, Q28

Diagram 11 is a pictogram which shows the sales of durians on Monday. The sales for Tuesday and Wednesday are not shown

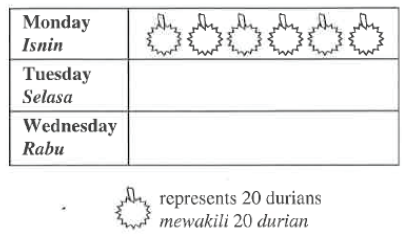


Diagram 11

The sales of durians on Monday, Tuesday and Wednesday are in the ratio 3:1:4. Find the total number of durians sold over the three days.

Informational & Representational : Mathematics 2009, Q28

Figure 2 Samples of problems with graphics containing dual-functions

Table 6 summarizes the percentages of graphics for each strand in both the SPM Mathematics and Additional Mathematics papers. Since we were looking for the percentages used for the topics involved, we calculated the percentages out of the total number of graphics employed for the whole respective papers. It could be seen from both distributions that the nature of the topics in their respective strands matched the percentages of questions that contained graphics. For example, topics such as polygon, transformation, solid geometry, lines and planes and earth as a sphere would definitely require some graphics to illustrate the situations rather than words alone which might create confusion or misinterpretation by the students or readers. About 27 – 30% of the problems with graphics were from the strand 2-Dimensional Problem which includes polygon, transformation, trigonometry, circles and angles of elevation and depression and bearing. The other strands of Functions and Graphs, 3-Dimensional Problem and Statistics contributed about 10 – 28% of the problems.

Table 6 The percentages of graphics in each strand in the SPM Mathematics papers

Topic/Year	2005	2006	2007	2008	2009	2010	2011	2012
Algebra	18	9	14	15	9	10	13	6
Functions & Graphs	18	19	24	18	25	28	23	19
2-Dimensional Problem	27	34	34	32	29	38	29	31
3-Dimensional Problem	21	19	17	18	19	17	16	22
Statistics	15	19	10	18	19	14	19	22

For the Additional Mathematics papers, as shown in Table 7, the biggest portion of 20 – 36% of the problems with graphics, specifically Cartesian or line graphs, were from the strand Functions and Graphs which contains topics such as functions, quadratics equations and functions, simultaneous equations and linear programming. The strand Coordinate Geometry and Vectors and Trigonometry which are also 'graphs in nature' contribute quite a significant portion of 13 – 23% and 15 - 23% respectively. Similar to the Mathematics papers, the strand Algebra of the Additional Mathematics papers did not need the presence of many graphics in their problems since the topics such as number base, algebraic expressions and formulae, linear inequalities, standard form, matrices, variations, mathematical reasoning, indices and logarithms, index numbers, progressions and linear law are mostly algebraic-prone.

Table 7 The percentages of graphics in each strand in the SPM Additional Mathematics papers

Topic/Year	2005	2006	2007	2008	2009	2010	2011	2012
Algebra	26	15	7	7	21	24	8	15
Functions & Graphs	21	20	36	33	29	24	23	23
Coordinate Geometry & Vectors	16	20	14	13	14	18	23	23
Trigonometry	16	15	21	20	21	18	23	23
Calculus	11	10	7	13	14	6	8	8
Statistics	11	20	14	13	0	12	15	8

In looking at the visual reasoning level demanded by the problems, the majority of the problems posed would just require students to read data directly from the graphics. Figure 3(a) illustrates a problem that requires students to read between the graphics or data embedded within the graphic. They have to make use and understand the connection between the isosceles RST, the right angle P and mathematical knowledge of the sum of the angles in a rectangle. Figure 3(b) is an example where students need to perform all the three levels of visual reasoning. Since the instructions for both parts (a)(i) and (ii) are 'state', in other words, no calculation is needed, students should be able to read off the latitude of Q and the location of R by just examining the graphic. Part (b) and (c) require students to scrutinize the graphic further and identify the relationship of the points together with mathematical knowledge on the properties of the sphere. Part (d) of the problem needs the students to go beyond their superficial thinking of the sphere and to critically and creatively apply visualization skills in order to solve for the average speed of the aeroplane's whole flight.

In Diagram 2, PQRST is a pentagon and RST is an isosceles triangle. PQV and STU are straight lines.

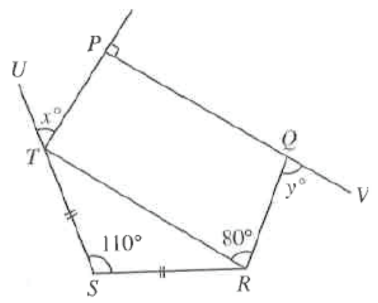


Diagram 2

Find the value of x + y.

Diagram 16 shows the locations of three points P, Q and R on the surface of the earth. O is the centre of the earth. The longitude of P is 95o E.

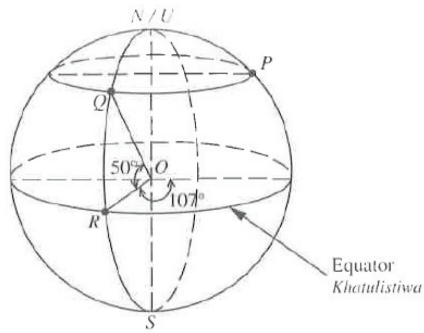


Diagram 16

a) (i) State the latitude of Q.
   (ii) State the location of R.
b) V lies 2460 miles due south of P. Calculate the latitude of V.
c) Calculate the distance, in nautical mile, from P due west to Q measured along the common parallel of latitude.
d) An aeroplane took off from P and flew due south to V. The time taken for the flight is 5 hours. Calculate the average speed, in knot, for the whole flight.

(a) /mReading between graphic: Mathematics 2010, Q8

(b) Reading beyond graphic : Mathematics 2009, Q16

Figure 3 Samples of problems with the required visual reasoning level.

DISCUSSION AND IMPLICATIONS

We have noticed that in the mathematics examination papers, the presence of graphics is numerous. The results of the study have implications on the teaching and learning and assessment of graphic-accompanied tasks within the school mathematics curriculum. Since the problem tasks were tracked from the national examination instruments, it is therefore vital to consider the results of the study from the assessment standpoint. How information is presented plays a critical role in establishing students' understanding of the mathematical concepts. Curriculum makers globally emphasized the teaching and learning of mathematics through graphics from as early as primary through tertiary levels (Monteiro & Ainley, 2010) so as to prepare students with skills and knowledge to read and interpret graphics. Generally, teachers and educators are officially advised to equip themselves and teach pupils and students beyond the level of simple reading and interpreting graphics but should be able to describe and relate the mathematical knowledge to real life situations.

Results of the study revealed variety in the nature and extent of graphics used in the mathematical problems. Educators and teachers may benefit from applying parts of the framework employed in the study to analyze the curriculum and consequently dismiss the impression that examination questions or problems are set and constructed alike. Graphic literacy is an elemental component of any student's development of mathematical understanding (NCTM, 2000). Graphics that are presented in the classroom are usually of the typical prototypes that require common procedural steps for solutions. This might cause confusion on the students upon encountering 'other' graphics not frequently exposed. In order to use graphics efficiently, students must be able to translate the word problems into graphics (this is the encoding process that complement the decoding process and not discussed in the study) and interpret the graphics to make meaning to the problems. One suggestion on innovative teaching of reading and interpreting graphics is through the development or improvement of the pedagogical activities that make use of the graphics from out-of-classroom contexts such as media publications: newspapers and magazines (Adler, 2000).

To sum up, from the study, it is evident that effective problem solving which makes use of graphics, depends greatly on the functions carried by the graphics together with the students' mathematical knowledge on the topic content and their visual reasoning ability. Therefore, it is very crucial that teachers and examination writers select appropriate graphics for particular function(s). Considering the critical needs for graphic literacy, we further suggest investigating students' methods of preference when solving mathematical problems in order to promote and encourage them to develop skills and strategies that they are inadequate of.

REFERENCES

Ainsworth, S., & Loizou, A. T. (2003). The effects of self-explaining when learning with text or diagrams. *Cognitive Science, 27,* 669-681.

Bardelle, C.(2010). Visual Proofs: An Experiment. Proceedings of CERME 6, January 28th-February 1st 2009, Lyon France. INRP 2010 <www.inrp.fr/editions/cerme6>

Blackwell, A., & Engelhardt, Y. (2002). A meta-taxonomy for diagram research. In M. Anderson, B. Meyer, & P. Olivier (Eds.), *Diagrammatic representation and reasoning,* 47–64. London: Springer-Verlag.

Carney, R. and Levin, J. R. (2002). Pictorial illustrations still improve students' learning from text. *Educational Psychology Review*, 14,1, 5- 26.

Cook, M. (2011). Teachers' use of visual representations in the science classroom. *Science Education International,* 22, 3,175-184.

Cox, R., & Grawemeyer, B. (2003). The mental organisation of external representations. *Proceedings of the European Cognitive Science Conference (EuroCogSci-joint Cognitive Science Society and German Cognitive Science Society conference), Osnabrück, September,2003.* Available from: http://www.cs.bath.ac.uk/~bg230/Cox&GrawemeyerEuroCogsci03.pdf

Curriculum Development Centre, Ministry of Education Malaysia (2003). *Integrated Curriculum for Secondary School Mathematics Curriculum Specifications.* Kuala Lumpur.

Dhakulkar & Nagarjuna, (2011) *An Analysis of Graphs in School Textbooks.* Proceedings of epiSTEME 4 International Conference to Review Research on Science, Technology and Mathematics Education, 127-131. India: Macmillan.

Diezmann, C. M. (1999) Assessing diagram quality: Making a difference to representation. In *Proceedings of the 22nd Annual Conference of Mathematics Education Research Group of Australasia*, 185-191, Adelaide.

Diezmann, C.M.& English, L. D..(2001) Promoting the use of diagrams as tools for thinking. In Cuoco, A.A. (ed.) *2001 National Council of Teachers of Mathematics Yearbook: The Role of Representation in School Mathematics.* National Council of Teachers of Mathematics, pp. 77-89.

Dufour-Janvier, B., Bednarz, N., & Belanger, M. (1987). Pedagogical considerations concerning the problem of representation. In C. Janvier (Ed.), *Problems of representation inthe teaching and learning of mathematics,* 110-120. Hillsdale, NJ: Erlbaum.

Francisco, J.M., & Maher, C.A. (2005). Conditions for promoting reasoning in problem solving: Insights from a longitudinal study. Journal of Mathematical Behavior, 24, 361-372.

Henningsen, M. & Stein, M.K.(1997). Mathematical tasks and student cognition: Classroom-based factors that support and inhibit high-level mathematical thinking and reasoning. Journal for Research in Mathematics Education, 28, 524-549.

Hembree, R. (1992). Experiments and relational studies in problem-solving: A meta-analysis. *Journal for Research in Mathematics Education, 23,* 242–273.

Ichikawa, S. (2000). *Benkyou-hou ga kawaru hon [A book about changing the approach to learning]* Tokyo: Iwanami Press. Larkin, J. H., & Simon, H. A. (1987). Why a diagram is (sometimes) worth ten thousand words. *Cognitive Science, 11,* 65-99.

Kidman,G. (2002). The Accuracy of mathematical diagrams in curriculum materials. In Cockburn, A. & Nardi, E. (Eds) *Proceeding of the PME* 26, 3, 201-208. UK: University of East Anglia.

Larkin, J. and Simon, H. (1987). Why a diagram is (sometimes) worth ten thousand words. *Cognitive Science*, 11:65-69.

Lowrie , T. & Diezmann, C. M. (2007). Middle School students' interpreting graphical tasks: Difficulties within a graphical language. *Proceeding 4th East Asia Regional Conference on Mathematics Education, Penang, Malaysia.*

Ministry of Education (2001). Malaysian Education Development Plan 2001-2010. Kuala Lumpur

Ministry of Education (2013). Malaysian Education Development Plan 2013-2025. Kuala Lumpur

Mayer, R.E. (2003). The promise of multimedia learning: Using the same instruction design methods across different media. *Learning and Instruction*, 13, 125-139.

Novick, L. R. (2006). Understanding spatial diagram structure: An analysis of hierarchies, matrices, and networks. *The Quarterly Journal of Experimental Psychology*, 59, 1826-1856.

Novick, L. R., & Hurley, S. M. (2001). To matrix, network, or hierarchy: That is the question. *Cognitive Psychology*, 42, 158–216.Organisation for Economic Co-operation and Development. (2004).Learning for tomorrow's world: First results from PISA 2003. Paris,VA: Author.

Pantziara, M., Gagatsis, A. &Pitta-Pantazi, D.(2004). The use of diagrams in solving non-routine problems.*Proceedings of the 28th Conference of the International Group for the Psychology of Mathematics Education*, 3, 489–496

Paoletti, G. (2004). *Writing-to-learn and graph drawing as aids for the integration of text and graphs.* In Rijlaarsdam, G., Van den Bergh, H. & Couzijn, M. (Eds.) Studies in writing. Volume 14, Effective learning and teaching of writing. Second Edition. Dordrecht: Kluwer Academic Publishers, 587-598.

Paivio, A. (1991). Dual coding theory: Retrospect and current status. *Canadian Journal of Psychology, 45*, 255–87.

Postigo, Y. & Pozo, J. (2004). On the road to graphicacy: The learning of graphical representation systems. *Educational Psychology*, 24(5), 623-644.

OECD (2005). *PISA 2003 Technical Report*, OECD, Paris.

Shaughnessy, J.M. (2003). Research on Students' Understanding of probability. In A Research Companion to Principles and Standards for School Mathematics. Kilpatrick, J., Martin, W.J., Schifter, D. (2003). (Eds.) Reston VA: *National Council of Teachers of Mathematics*, 216-226.

Uesaka, Y., & Manalo, E. (2011). The effects of peer communication with diagrams on students' math word problem solving processes and outcomes. In L. Carlson, C. Hoelscher, & T. Shipley (eds.), *Proceedings of the 33rd Annual Conference of the Cognitive Science Society,*312–317. Austin, TX: Cognitive Science Society.

Uesaka, Y., Manalo, E., & Ichikawa, S. (2007). What kinds of perceptions and daily learning behaviours promote students' use of diagrams in mathematics problem solving? *Learning and Instruction,*17, 322–335.

Uesaka, Y., Manalo, E., & Ichikawa, S. (2010). The effects of perception of efficacy and diagram construction skills on students' spontaneous use of diagrams when solving math word problems. In A. K. Goel, M. Jamnik,& N. H. Narayanan (eds.), Diagrams 2010, LNAI 6170 (pp. 197–211).Berlin, Heidelberg: Springer-Verlag.

Winn, B. (1987). Charts, graphs, and diagrams in educational materials. In D. M.Willows & H. A. Houghton (Eds.). *The psychology of illustration*, 1. New York:Springer-Verlag.