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# Assessment of Pre-Service Secondary Mathematics Teachers' Van Hiele Levels of Geometric Thinking

**Chew Cheng Meng**
*Universiti Sains Malaysia*
**Noraini Idris**
*Universiti Pendidikan Sultan Idris*

## Abstract

*The purpose of this study was to assess pre-service secondary mathematics teachers' van Hiele levels of geometric thinking using the Van Hiele Geometry Test. The 25-item, multiple-choice, paper-and-pencil test was developed by the Cognitive Development and Achievement in Secondary School Geometry Project based on the van Hiele Theory of Geometric Thinking. The participants comprised 147 pre-service secondary mathematics teachers who attended a mathematics teaching methods course in a Malaysian public university. The data were analysed based on the '4 of 5 criterion' to minimise the chance of a student being at a level by guessing. The results showed that 16 (10.9%) of the participants were at Level 0, 52 (35.4%) were at Level 1, 62 (42.2%) were at Level 2, 9 (6.1%) were at Level 3, 1 (0.7%) were at Level 4, none (0.0%) was at Level 5, and 7 (4.8%) could not be assigned a van Hiele level because their responses did not fit the '4 of 5 criterion.' Most of the participants were at or below van Hiele Level 2. Very few participants were at van Hiele Level 3 which is the minimum level for pre-service secondary mathematics teachers to teach geometry because the Malaysian secondary school geometry contents are up to Level 3.*

***Keywords*** *Van Hiele levels of geometric thinking, Van Hiele Geometry Test, Pre-service secondary mathematics teachers*

## INTRODUCTION

Geometry is recognized as a basic skill in mathematics (Hoffer & Hoffer, 1992; National Council of Supervisors of Mathematics, 1977; National Council of Teachers of Mathematics, 1989, 2000) because it has important applications to topics in basic mathematics and gives valuable preparation for courses in higher mathematics and the sciences as well as for a variety of careers requiring mathematical skills. In addition, geometry has important applications to real-life problems.

However, in spite of its importance, the performance of Malaysian secondary school students in geometry was still discouraging as highlighted in the Trends in International Mathematics and Science Study (TIMSS 2007). Specifically, in the TIMSS 2007 Report, the average geometry achievement of Malaysian Form Two students (that is second-year secondary school students of ages 14 or 15 years old) was not only significantly lower than the TIMSS scale average but also far below the average geometry achievement of the top five Asia-Pacific countries of Chinese Taipei, Republic of Korea, Singapore, Japan and Hong Kong SAR as shown in Table 1 (Mullis, Martin, & Foy, 2008).

**Table 1** *The international ranking in geometry achievement in TIMSS 2007*

| International ranking | Country | Average scale score in geometry |
|---|---|---|
| First | Chinese Taipei | 592* |
| Second | Republic of Korea | 587* |
| Third | Singapore | 578* |
| Fourth | Japan | 573* |
| Fifth | Hong Kong SAR | 570* |
| Twenty-fourth | Malaysia | 477 |

*Significantly higher than TIMSS scale average (500).

As a result, Malaysia was ranked twenty-fourth in geometry achievement out of 49 participating countries and 7 benchmarking participants in the TIMSS 2007. The low ranking indirectly reflected that our Malaysian Form Two students' levels of geometric thinking were still far from satisfactory. In addressing this concern, it is important that secondary school students, particularly lower secondary school students, are taught by mathematics teachers who have high levels of geometric thinking. However, to what extent is the geometric thinking level of Malaysian mathematics teachers, especially the pre-service secondary mathematics teachers? Therefore, it is necessary to assess pre-service secondary mathematics teachers' levels of geometric thinking.

## PURPOSE OF THE STUDY

The purpose of this study was to assess pre-service secondary mathematics teachers' van Hiele levels of geometric thinking. More specifically, this study aimed to address the following research question: What were the pre-service secondary mathematics teachers' van Hiele levels of geometric thinking?

## THEORETICAL FRAMEWORK

The theory underpinning this study is the van Hiele Theory of Geometric Thinking. According to the theory, students progress sequentially through five hierarchical levels of thinking in the process of learning geometry (Usiskin, 1982, p. 77):

Level 1 (Recognition) - the student can learn names of figures and recognizes a shape as a whole.
Level 2 (Analysis) - the student can identify properties of figures.
Level 3 (Order) - the student can logically order figures and relationships, but does not operate within a mathematical system.

Level 4 (Deduction) - the student understands the significance of deduction and the roles of postulates, theorems and proof.
Level 5 (Rigor) - the student understands the necessity for rigor and is able to make abstract deductions.

Furthermore, Clements and Battista (1992) proposed the existence of Level 0 (Pre-recognition). At this level of geometric thinking, the student notices only a subset of the visual characteristics of a shape, resulting in an inability to distinguish between figures (Mason, 1997).

## METHODOLOGY

### Research design and sample

The researchers employed a cross-sectional survey which involved collecting data from selected pre-service secondary mathematics teachers in a single time period (Gay & Airasian, 2003). The sample of this study consisted of 147 pre-service secondary mathematics teachers enrolled under two different programs in a Malaysian public university, namely 137 participants in Bachelor of Science (Education) and 10 participants in Bachelor of Education (Science). Their age ranged from 21 to 25 years old. The participants comprised 117 females and 30 males. They attended a mathematics teaching methods course and no geometry lesson was given to them before taking the course. In addition, none of the participants had taken a formal course in geometry prior to taking the mathematics teaching methods course.

### Research procedure and instrument

During the first lecture of the mathematics teaching methods course, the first author administered The Van Hiele Geometry Test (VHGT) to all the participants to assess their van Hiele levels of geometric thinking. The VHGT was developed by the Cognitive Development and Achievement in Secondary School Geometry (CDASSG) project at the University of Chicago based on the van Hiele theory (Usiskin, 1982). It was a 25-item multiple-choice, paper-and-pencil test with five options per item and five items per van Hiele level of geometric thinking. It contained five subtests corresponding to five different van Hiele levels of geometric thinking as shown in Table 2.

**Table 2** Distribution of the VHGT items

| Item number | van Hiele level |
|---|---|
| 1 - 5 | Level 1 |
| 6 - 10 | Level 2 |
| 11 - 15 | Level 3 |
| 16 - 20 | Level 4 |
| 21 - 25 | Level 5 |

The VHGT required about 35 minutes to complete. Figures 1 through 5 show the sample items from the five subtests of the VHGT, respectively (Usiskin, 1982).

1. Which of these are squares?

(A) K only
(B) L only
(C) M only
(D) L and M only
(E) All are squares.

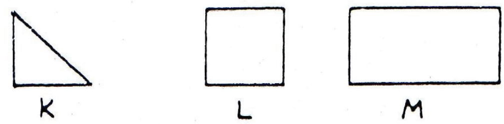



**Figure 1** Sample item from the Level 1 subtest.

6. PQRS is a square.

Which relationship is true in all squares?

(A) $\overline{PR}$ and $\overline{RS}$ have the same length.
(B) $\overline{QS}$ and $\overline{PR}$ are perpendicular.
(C) $\overline{PS}$ and $\overline{QR}$ are perpendicular.
(D) $\overline{PS}$ and $\overline{QS}$ have the same length.
(E) Angle Q is larger than angle R.

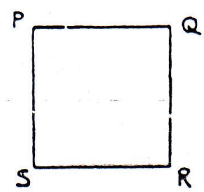



**Figure 2** Sample item from the Level 2 subtest.

11. Here are two statements.

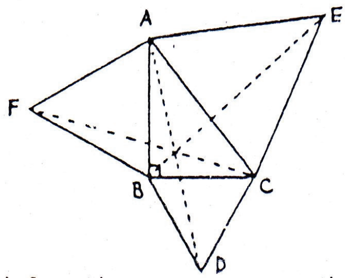
Statement 1: Figure F is a rectangle.
Statement 2: Figure F is a triangle.

Which is correct?

(A) If 1 is true, then 2 is true.
(B) If 1 is false, then 2 is true.
(C) 1 and 2 cannot both be true.
(D) 1 and 2 cannot both be false.
(E) None of (A)-(D) is correct.

**Figure 3** Sample item from the Level 3 subtest.

16. Here is a right triangle ABC. Equilateral triangles ACE, ABF, and BCD have been constructed on the sides of ABC.




From this information, one can prove that $\overline{AD}$, $\overline{BE}$, and $\overline{CF}$ have a point in common. What would this proof tell you?

(A) Only in this triangle drawn can we be sure that $\overline{AD}$, $\overline{BE}$ and $\overline{CF}$ have a point in common.

(B) In some but not all right triangles, $\overline{AD}$, $\overline{BE}$ and $\overline{CF}$ have a point in common.

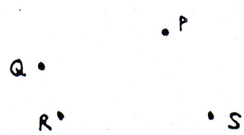
(C) In any right triangle, $\overline{AD}$, $\overline{BE}$ and $\overline{CF}$ have a point in common.

(D) In any triangle, $\overline{AD}$, $\overline{BE}$ and $\overline{CF}$ have a point in common.

(E) In any equilateral triangle, $\overline{AD}$, $\overline{BE}$ and $\overline{CF}$ have a point in common.

**Figure 4** Sample item from the Level 4 subtest.

21. In F-geometry, one that is different from the one you are used to, there are exactly four points and six lines. Every line contains exactly two points. If the points are P, Q, R, and S, the lines are
{P,Q} , {P,R} ; {P,S} , {Q,R} , {Q,S} , and {R,S}




Here are how the words "intersect" and "parallel" are used in F-geometry. The lines {P,Q} and {P,R} intersect at P because {P,Q} and {P,R} have P in common.

The lines {P,Q} and {R,S} are parallel because they have no points in common.

From this information, which is correct?

(A) {P,R} and {Q,S} intersect.

(B) {P,R} and {Q,S} are parallel.

(C) {Q,R} and {R,S} are parallel.

(D) {P,S} and {Q,R} intersect.

(E) None of (A)-(D) is correct.

**Figure 5** Sample item from the Level 5 subtest.

Usiskin (1982) used two different scoring criteria in the CDASSG project, either 3 of 5 correct or 4 of 5 correct for each subtest. The researchers employed the '4 of 5' criterion to minimise the chance of a participant being at a level by guessing (Mason, 1997). This means that if a participant answered 4 of 5 items correctly in a given subtest, he or she was considered to have mastered that level of geometric thinking. If a participant met the criterion for mastery of level up to and including Level n and failed to meet the criterion for mastery of all the levels above Level n, the participant was assigned to Level n. If a participant, for example, correctly answered 5, 4, 3, 2 and 1 items out of five Level-1, Level-2, Level-3, Level-4, and Level-5 items respectively, then he or she would be assigned to Level-2 geometric thinking. If the participant could not be assigned to a level in this manner, the participant was said to "not fit" (Mason, 1997; Usiskin, 1982). For example, if a participant correctly answered 5, 3, 4, 2 and 1 items out of five Level-1, Level-2, Level-3, Level-4, and Level-5 items respectively, then he or she was said to "not fit."

## RESULTS

Table 3 shows the patterns of correct responses, frequency and percentage of the pre-service secondary mathematics teachers operating at each van Hiele level of geometric thinking. The letters, a, b, c, d and e in the patterns of correct responses, [abcde] indicate the number of correct responses for Level-1, Level-2, Level-3, Level-4 and Level-5 items, respectively. For instance, the pattern of correct responses, [23002] for a participant operating at van Hiele Level 0 indicates that he or she has 2 correct responses for Level-1 items, 3 correct responses for Level-2 items, 0 correct response for Level-3 items, 0 correct response for Level-4 items, and 2 correct responses for Level-5 items, respectively. Likewise, the pattern of correct responses, [55543] for a participant operating at van Hiele Level 4 indicates that he or she has 5 correct responses for Level-1 items, 5 correct responses for Level-2 items, 5 correct responses for Level-3 items, 4 correct responses for Level-4 items, and 3 correct responses for Level-5 items, respectively. However, the pattern of correct responses, [41410] for a participant who was classified as "not fit" indicates that he or she has 4 correct responses for Level-1 items, 1 correct response for Level-2 items, 4 correct response for Level-3 items, 1 correct response for Level-4 items, and 0 correct response for Level-5 items, respectively.

As illustrated in Table 3, 16 (10.9%) of the participants were at Level 0 (notice only a subset of the visual characteristics of a shape, resulting in an inability to distinguish between figures); 52 (35.4%) were at Level 1 (can learn names of figures and recognizes a shape as a whole); 62 (42.2%) were at Level 2 (can identify properties of figures); 9 (6.1%) were at Level 3 (can logically order figures and relationships, but does not operate within a mathematical system); 1 (0.7%) was at Level 4 (understand the significance of deduction and the roles of postulates, theorems and proof); none (0%) was at Level 5 (understand the necessity for rigor and is able to make abstract deductions); and 7 (4.8%) could not be assigned a van Hiele level because their responses did not fit the '4 of 5 criterion.'

**Table 3** Patterns of correct responses, frequency and percentage of participants at each van Hiele level

| van Hiele Level | Patterns of correct responses, [abcde] | Frequency | Percentage |
|---|---|---|---|
| 0 | [22122], [23002], [23101], [23111], [31121], [31212], [31312], [32111], [32120], [33010], [33111], [33122], [33211], [33221], [33302], [33313], | 16 | 10.9 |
| 1 | [41011], [41210], [42031], [42101], [42101], [42200], [42222], [42231], [42301], [42321], [43001], [43001], [43120], [43121], [43121], [43122], [43201], [43210], [43210], [43211], [43211], [43212], [43221], [43301], [43310], [43312], [43321], [51221], [52102], [52102], [52113], [52201], [52202], [52212], [52221], [52223], [52302], [53100], [53102], [53110], [53111], [53112], [53201], [53211], [53211], [53211], [53212], [53223], [53311], [53311], [53313], [53320], | 52 | 35.4 |
| 2 | [44100], [44111], [44111], [44132], [44201], [44201], [44210], [44211], [44211], [44212], [44212], [44221], [44231], [44301], [44313], [44313], [44313], [44320], [44320], [44321], [45103], [45121], [45200], [45221], [45300], [45301], [45302], [45302], [45312], [45321], [45323], [54000], [54010], [54101], [54113], [54122], [54200], [54200], [54203], [54211], [54211], [54213], [54223], [54302], [54312], [54323], [55123], [55200], [55201], [55201], [55202], [55210], [55211], [55211], [55222], [55222], [55231], [55300], [55310], [55320], [55322], [55322] | 62 | 42.2 |
| 3 | [44402], [44413], [44422], [45521], [54421], [54422], [55421], [55421],[55512] | 9 | 6.1 |
| 4 | [55543] | 1 | 0.7 |
| 5 | | 0 | 0.0 |
| Not fit | [41410], [43410], [54104], [54105], [54314], [54414], [54425] | 7 | 4.8 |
| Total | | 147 | 100.0 |

## DISCUSSION AND CONCLUSION

The results indicated that the pre-service secondary mathematics teachers' van Hiele levels of geometric thinking ranged from Level 0 to Level 4 in different frequencies and percentages. In particular, 16 (10.9%) of the participants were at Level 0 and 52 (35.4%) were at Level 1. This implies that 68 (46.3%) of the participants lacked higher levels of geometric thinking to teach secondary school geometry as they are expected to teach geometry to secondary school students who are supposed to attain at least Level 2 of geometric thinking or above. In other words, secondary school students should be able to identify properties of figures (Level 2) such as polygons, circles and geometric solids. They should also understand definitions of concepts like squares, rectangles, cubes, cuboids, prisms, pyramids, cylinders and cones (Level 3) (Malaysian Ministry of Education, 2003a, 2003b, 2003c).

Moreover, only 10 (6.8%) of the pre-service secondary mathematics teachers' van Hiele levels of geometric thinking were at or above Level 3. This result is in sharp contrast to the result of Halat's (2008) study. He found that 42.3% of the pre-service secondary mathematics teachers' van Hiele levels were at or above Level 3. While Halat found that 1.9% of the participants attained van Hiele Level 5, none of the participants of this study attained van Hiele Level 5. Nevertheless, this finding is still consistent with the finding of Knight's (2006) study. He found that the pre-service secondary mathematics teachers' van Hiele levels of geometric thinking were below Level 4.

Since the Malaysian secondary school geometry contents are up to van Hiele Level 3 (Malaysian Ministry of Education, 2003a, 2003b, 2003c), the attainment of van Hiele Level 3 should be the minimum goal for all the pre-service secondary mathematics teachers. But, the results showed that 130 (88.5%) of the pre-service secondary mathematics teachers' van Hiele levels of geometric thinking were at or below van Hiele Level 2. Hence, these pre-service secondary mathematics teachers need to enhance their levels of geometric thinking so that they can help their future students to develop higher levels of geometric thinking.

To help students progress from one level of geometric thinking to the next, the van Hieles propose a sequence of five phases of learning or phase-based instruction, namely inquiry or information, guided orientation, explicitation, free orientation, and integration (van Hiele, 1986; 1959/1984; 1999; van Hiele-Geldof, 1959/1984). In fact, research has shown that the incorporation of the five phases of learning or phase-based instruction in geometry teaching and learning could enhance the van Hiele levels of geometric thinking of elementary in-service teachers (McClendon, 1990), prospective elementary school teachers (Wu, 1994), high school students (Bobango, 1987), middle-grade or secondary school students (Baynes, 1999; Breen, 2000; Chew, 2007; Chew & Noraini, 2007; Choi, 1996; Choi-Koh, 1999; Fuys & Geddes, 1984; Fuys, Geddes, Lovett & Tischler, 1988; Jaimie-Pastor, 1995; Massey, 1993; Moran, 1993; Tay, 2003; van Hiele-Geldof, 1959/1984), elementary school students (Chew & Lim, 2010; Matthews, 2005), and kindergartner students (Dye, 1991). This implies that mathematics teacher education should provide a special geometry program that incorporates the five phases of learning in order to help pre-service secondary mathematics teachers to attain higher levels of geometric thinking, especially van Hiele Level 3.

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